Winter term 2015/16
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Combinatorial Optimization

Exercise Sheet 6

Exercise 6.1: Prove:

(i) For every two-edge-connected graph G and every $T \subseteq V(G)$ with $|T|$ even there exists a T-join J in G with $|J| \leq \frac{|E(G)|}{2}$. (2 Points)

(ii) For every factor-critical graph G and every $T \subseteq V(G)$ with $|T|$ even there exists a T-join J in G with $|J| \leq \frac{|V(G)|-1}{2}$. (2 Points)

Exercise 6.2: For $n \in \mathbb{N}$, let P_n be the convex hull of all even 0-1-vectors. More precisely, let

$$P_n = \operatorname{conv}\{x \in \{0,1\}^n : \sum_{i=0}^{n} x_i \equiv 0 \pmod 2\}.$$

Prove that the relaxation complexity is of size $rc(P_n) = 2^{\Theta(n)}$. (4 Points)

Exercise 6.3: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the following inequality system with $\mathcal{O}(n^3)$ variables and constraints describes a polytope whose orthogonal projection to the x-variables yields the spanning tree polytope of G.

$$\begin{aligned}
x_e &\geq 0 && (e \in E)\\
z_{u,v,w} &\geq 0 && (\{u,v\} \in E, w \in V \setminus \{u,v\})\\
\sum_{e \in E} x_e &= n - 1 &&\\
x_e &= z_{u,v,w} + z_{v,u,w} && (e = \{u,v\} \in E, w \in V \setminus e)\\
x_e + \sum_{\{u,v\} \in \delta(v) \setminus \{e\}} z_{u,v,w} &= 1 && (v \in V, e = \{v,w\} \in \delta(v))
\end{aligned}$$

(4 Points)

Exercise 6.4: Show that the following algorithm finds in a graph G (which is not a forest) with edge weights $w : E(G) \to \mathbb{R}$ a cycle $C \subset E(G)$ that minimizes $\frac{w(C)}{|C|}$ in strongly polynomial time: First reduce all edge weights by $\max\{w(e)|e \in E(G)\}$. Then find a minimum-weight $\emptyset$-join J. If $w(J) = 0$ output a cycle of weight 0, otherwise add $\frac{-w(J)}{|J|}$ to all edge weights and iterate (i.e. find again a minimum-weight $\emptyset$-join). (4 Points)

Deadline: Tuesday, December 8, 2015, **before** the lecture.
Information: Submissions by groups of up to **three** students are allowed.