Exercise Sheet 4

Exercise 4.1: Let $G = (V, E)$ be a graph. A set $\mathcal{H} = \{S_1, \ldots, S_k, v_1, \ldots v_r\}$ has property A if

- $S_i \subseteq V$ and $|S_i|$ is odd for $1 \leq i \leq k$,
- $v_i \in V$ for $1 \leq i \leq r$, and
- for each $e \in E$ either $e \subseteq S_i$ for some $i \in \{1, \ldots, k\}$ or $v_i \in e$ for some $i \in \{1, \ldots, r\}$.

The weight of a set $\mathcal{H}$ with property A is $w(\mathcal{H}) := r + \sum_{i=1}^{k} \frac{|S_i| - 1}{2}$. Prove $\nu(G) = \min\{w(\mathcal{H}) \mid \mathcal{H}$ is a set with property A\}$.

(4 Points)

Exercise 4.2: Let $G = (V, E)$ be a $k$-connected graph with $2\nu(G) < |V| - 1$.

(i) Prove $\nu(G) \geq k$. (2 Points)

(ii) Prove $\tau(G) \leq 2\nu(G) - k$. (2 Points)

Exercise 4.3: Given an undirected graph $G$, an edge is called unmatchable if it is not contained in any perfect matching. How can one determine the set of unmatchable edges in $O(n^3)$ time? (4 Points)

Hint: First determine a perfect matching in $G$. Then determine for each vertex $v$ the set of unmatchable edges incident to $v$.

Exercise 4.4: Show how the following problem can be solved in polynomial time: Given a graph $G$ and edge weights $c : E(G) \to \mathbb{R}_{>0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. (4 Points)

Deadline: Tuesday, November 24, 2015, before the lecture.