Winter term 2015/16
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Combinatorial Optimization

Exercise Sheet 4

Exercise 4.1: Let $G = (V, E)$ be a graph. A set $\mathcal{H} = \{S_1, \dots, S_k, v_1, \dots v_r\}$ has property A if

- $S_i \subseteq V$ and $|S_i|$ is odd for $1 \leq i \leq k$,
- $v_i \in V$ for $1 \leq i \leq r$, and
- for each $e \in E$ either $e \subseteq S_i$ for some $i \in \{1, \dots, k\}$ or $v_i \in e$ for some $i \in \{1, \dots, r\}$.

The weight of a set $\mathcal{H}$ with property A is $w(\mathcal{H}) := r + \sum_{i=1}^{k} \frac{|S_i| - 1}{2}$. Prove

$$\nu(G) = \min\{w(\mathcal{H}) \,|\, \mathcal{H} \text{ is a set with property A}\}.$$

(4 Points)

Exercise 4.2: Let $G = (V, E)$ be a k-connected graph with $2\nu(G) < |V| - 1$.

(i) Prove $\nu(G) \geq k$. (2 Points)

(ii) Prove $\tau(G) \leq 2\nu(G) - k$. (2 Points)

Exercise 4.3: Given an undirected graph G, an edge is called *unmatchable* if it is not contained in any perfect matching. How can one determine the set of unmatchable edges in $\mathcal{O}(n^3)$ time? (4 Points)
Hint: First determine a perfect matching in G. Then determine for each vertex v the set of unmatchable edges incident to v.

Exercise 4.4: Show how the following problem can be solved in polynomial time: Given a graph G and edge weights $c : E(G) \to \mathbb{R}_{>0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. (4 Points)

Deadline: Tuesday, November 24, 2015, before the lecture.