Winter term 2013/14
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Combinatorial Optimization

Exercise Sheet 10

Exercise 10.1:
Consider the TSP on n cities. For any weight funcion $w : E(K_n) \to \mathbb{R}_+$ let c_w^* be the length of an optimum tour with respect to w. Prove: If $L_1 \leq c_{w_1}^*$ and $L_2 \leq c_{w_2}^*$ for two weight functions w_1 and w_2 then also $L_1 + L_2 \leq c_{w_1+w_2}^*$, where the sum of the two weight functions is taken componentwise.

(2 points)

Exercise 10.2:
Let $x \in [0,1]^{E(K_n)}$ with $\sum_{e\in\delta(v)} x_e = 2$ for all $v \in V(K_n)$. Prove that if there exists a violated subtour contraint, i.e a set S with $\emptyset \neq S \subset V(K_n)$ and $\sum_{e\in\delta(S)} x_e < 2$ then there exists one with $x_e < 1$ for all $e \in \delta(S)$.

(4 points)

Exercise 10.3:
Consider the TSP in the two-dimensional Euclidean plane. Let T be a tour. Prove that all edge crossings can be removed in $O(n^3)$ time from T by taking two crossing edges, say $\{A, C\}$ and $\{B, D\}$, remove the crossing (that is to say remove $\{A, C\}$ and $\{B, D\}$ from the tour and insert either $\{A, B\}$ and $\{C, D\}$ or $\{A, D\}$ and $\{B, C\}$ such that we still have a tour) and iterate.
Hint: For each edge, viewed as a line segment, count the number of (infinite) lines that can be drawn through two cities so as to intersect that segment. Show that the removal of a crossing reduces the total count, for all edges, by at least one.

(4 points)

Exercise 10.4:
Prove that every edge in a 3-regular graph is contained in an even number of Hamiltonian circuits.

(6 points)

Deadline: Thursday, January 16, 2014, before the lecture.

Merry Christmas and a happy new year!