

## Combinatorial Optimization

### Exercise Sheet 10

#### Exercise 10.1:

Consider the TSP on  $n$  cities. For any weight function  $w : E(K_n) \rightarrow \mathbb{R}_+$  let  $c_w^*$  be the length of an optimum tour with respect to  $w$ . Prove: If  $L_1 \leq c_{w_1}^*$  and  $L_2 \leq c_{w_2}^*$  for two weight functions  $w_1$  and  $w_2$  then also  $L_1 + L_2 \leq c_{w_1+w_2}^*$ , where the sum of the two weight functions is taken componentwise.

(2 points)

#### Exercise 10.2:

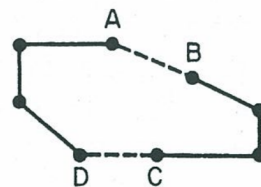
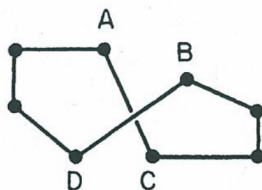
Let  $x \in [0, 1]^{E(K_n)}$  with  $\sum_{e \in \delta(v)} x_e = 2$  for all  $v \in V(K_n)$ . Prove that if there exists a violated subtour constraint, i.e. a set  $S$  with  $\emptyset \neq S \subset V(K_n)$  and  $\sum_{e \in \delta(S)} x_e < 2$  then there exists one with  $x_e < 1$  for all  $e \in \delta(S)$ .

(4 points)

#### Exercise 10.3:

Consider the TSP in the two-dimensional Euclidean plane. Let  $T$  be a tour. Prove that all edge crossings can be removed in  $O(n^3)$  time from  $T$  by taking two crossing edges, say  $\{A, C\}$  and  $\{B, D\}$ , remove the crossing (that is to say remove  $\{A, C\}$  and  $\{B, D\}$  from the tour and insert either  $\{A, B\}$  and  $\{C, D\}$  or  $\{A, D\}$  and  $\{B, C\}$  such that we still have a tour) and iterate.

*Hint:* For each edge, viewed as a line segment, count the number of (infinite) lines that can be drawn through two cities so as to intersect that segment. Show that the removal of a crossing reduces the total count, for all edges, by at least one.



(4 points)

**Exercise 10.4:**

Prove that every edge in a 3-regular graph is contained in an even number of Hamiltonian circuits.

(6 points)

**Deadline:** Thursday, January 16, 2014, before the lecture.

**Merry Christmas and a happy new year!**