Exercises 6

Exercise 1:

An **odd set cover** of a graph $G$ is a set $\mathcal{H} = \{S_1, \ldots, S_k, v_1, \ldots, v_r\}$ of subsets $S_i \subseteq V(G)$ of odd cardinality and vertices $v_i \in V(G)$ such that for each edge $e \in E(G)$ either both endpoints of $e$ are contained in one of the $S_i$’s or $e$ is incident to one of the $v_i$’s.

The **weight** of an odd set cover $\mathcal{H}$ is $w(\mathcal{H}) := r + \sum_{i=1}^k |S_i| - \frac{1}{2} |S_i| - 1$.

Prove the following generalization of König’s Theorem for general graphs:

$$\nu(G) = \min\{w(\mathcal{H}) \mid \mathcal{H} \text{ odd set cover of } G\}$$

for any graph $G$.

(4 points)

Exercise 2:

Let $G = (V, E)$ be a graph, $c : E \to \mathbb{R}$ and $M$ a matching in $G$ with $c(M) > 0$. Consider an $M$-augmenting path (or cycle) $P$ and its **relative gain** $\text{gain}_{\text{rel}}(P) := \frac{c(M \Delta P)}{c(M)}$. Recall that $\text{aug}(v)$ (the maximum gain 2-augmentation centered at $v$) can be obtained in linear time in $\deg(v) + \deg(\mu(v))$. What is the fastest algorithm to compute a maximum relative gain 2-augmentation centered at $v$?

(4 points)

Exercise 3:

Let $G$ be a graph. Show that a minimum edge cover in $G$ can be computed in polynomial time.

(4 points)

Deadline: Tuesday, November 23rd, before the lecture.