Exercises 10

Exercise 1:
Compute (sharp) lower bounds for the rank quotients of the following independence systems \((E,F)\):

(a) \(E = V(G)\) and \(F\) stable sets in \(G\).
(b) \(E = E(G)\) and \(F\) are subsets of a Hamiltonian circle in \(G\).
(c) \(E = E(G)\) and \(F\) are subsets of an \(s-t\)-path with \(s, t \in V(G)\) and \(t\) reachable from \(s\).
(d) \(E = \{1, \ldots, n\}\) and non-negative weights \(w_j, j = 1, \ldots, n\) and \(k \in \mathbb{R}_+\). \(F\) are subsets of total weight \(\leq k\).
(e) \(E = E(G), F\) are subsets of Steiner trees in \(G\).
(f) \(E = E(G), F\) are subsets of branchings in \(G\).
(g) \(E = E(G), F\) contains matchings in \(G\).

(6 points)

Exercise 2:
Show that the following decision problem is \(\mathcal{NP}\)-complete: Given three matroids \((E,F_1), (E,F_2), (E,F_3)\) (by some oracle) and a \(k \in \mathbb{N}\). Exists an \(F\) in \(F_1 \cap F_2 \cap F_3\) such that \(|F| \geq k\) ?

(3 points)

Exercise 3:
Let \(\mathcal{M}\) be a graphic matroid. Prove that there exists a connected graph \(G\) such that
\[
\mathcal{M} \cong \mathcal{M}(G)
\]

(4 points)

Exercise 4:
Let \(G\) be an undirected graph, \(k \in \mathbb{N}\) and let \(\mathcal{F} := \{F \subseteq E(G) | F \text{ is union of } k \text{ forests} \}\).
Prove: \((E(G), \mathcal{F})\) is a matroid.

(3 points)

Deadline: Tuesday, December 21st, before the lecture.