Exercise Set 11

Exercise 11.1. Show that the ratio of the cost of the integral primal solution and the cost of the dual solution computed by the primal-dual algorithm for the Prize-Collecting Steiner Forest Problem discussed in the lecture (which yields a 3-approximation) can get arbitrarily close to 3.

Hint: Consider a grid-graph with vertex set $\{v_{i,j} : 1 \leq i \leq 2, 1 \leq j \leq \ell\}$ *of height* 2 *and width* $\ell \gg 1$. *As vertex pairs (for which a penalty is charged in case they are not connected), choose* $\{v_{i,1}, v_{i,\ell}\}$ *for* $i = 1, 2$ *and* $\{v_{1,j}, v_{2,j}\}$ *for* $2 \leq j \leq \ell - 1$. *Choose appropriate edge weights and penalties.*

(5 points)

Exercise 11.2. Show that for each $k \in \mathbb{N}$, there is an instance of the Prize-Collecting Steiner Forest Problem with $|\mathcal{P}| = k$ ($\mathcal{P}$ denotes the set of vertex pairs for which a penalty is charged in case they are not connected in the computed solution) such that the 2-approximation algorithm discussed in the lecture takes k many iterations/recursive calls.

Hint: Consider a path and choose edge weights, penalties, and $\mathcal{P}$ *appropriately.*

(5 points)

Exercise 11.3. Consider the restriction $\mathcal{P}$ of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant B.

Let $\varepsilon > 0$. Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1+\epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B + 1)\epsilon$.

(5 points)

Exercise 11.4. Let $G = (V, E)$ be an undirected graph. For a partition $\mathcal{P}$ of the vertex set V let

$$\delta(\mathcal{P}) := \{e : e \in \delta(U) \text{ for some } U \in \mathcal{P}\}.$$

Prove

$$\left\{ x \in [0,1]^E : \sum_{e \in E} x_e = |V(G)| - 1, \sum_{e \in E(G[X])} x_e \leq |X| - 1 \text{ for } \emptyset \neq X \subsetneq V(G) \right\}$$

$$= \left\{ x \in [0,1]^E : \sum_{e \in E} x_e = |V(G)| - 1, \sum_{e \in \delta(\mathcal{P})} x_e \geq |\mathcal{P}| - 1 \text{ for every partition } \mathcal{P} \text{ of } V \right\}.$$

(5 points)

Deadline: Tuesday, July 2^{nd}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/appr_ss24_ex.html`

In case of any questions feel free to contact me at blauth@or.uni-bonn.de.