Exercise Set 8

Exercise 8.1. Consider the following algorithm for the STEINER TREE PROBLEM with 3 terminals v_1, v_2 and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

(i) $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log(|V|))$ time and works correctly.

(5 points)

Exercise 8.2. Prove that the 4-Steiner ratio ρ_4 is $\frac{3}{2}$.

(5 points)

Exercise 8.3. Show that for any terminal spanning tree (T, S) and any k-component X, the set

$$\mathcal{B} := \{D \subseteq S : (S \setminus D) \cup E(X) \text{ is the edge set of a connector for } T\}$$

is the set of independent sets of a matroid.

(5 points)

Exercise 8.4. Let T denote the terminal set in the STEINER TREE PROBLEM and let $r \in T$ be an arbitrarily chosen root. Let

$$\text{LP} = \min \left\{ c(x) : \sum_{e \in \delta(U)} x_e \geq 1 \text{ for } U \subseteq V \setminus \{r\} \text{ with } U \cap T \neq \emptyset, x \geq 0 \right\}.$$

Now we replace every edge $\{v, w\}$ by two directed edges (v, w) and (w, v) (with cost $c(\{v, w\})$. Consider the following LP:

$$\text{BCR} = \min \left\{ c(x) : \sum_{e \in \delta^-(U)} x_e \geq 1 \text{ for } U \subseteq V \setminus \{r\} \text{ with } U \cap T \neq \emptyset, x \geq 0 \right\}.$$

(a) Prove that the value BCR is independent of the choice of the root $r \in T$.

(b) Show that the supremum of $\frac{\text{BCR}}{\text{LP}}$ over all instances (with $\text{LP} \neq 0$) is at least 2.

(5 points)

Deadline: Tuesday, June 11^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/appr_ss24_ex.html`

In case of any questions feel free to contact me at blauth@or.uni-bonn.de.