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Exercise Set 5

Exercise 5.1. Let $A = (a_i)_{1 \leq i \leq p}$ and $B = (b_j)_{1 \leq j \leq q}$ be two inputs of the Bin Packing problem. We write $A \subseteq B$ if there are indices $1 \leq k_1 < k_2 < \cdots < k_p \leq q$ with $a_i \leq b_{k_i}$ for $1 \leq i \leq p$. An algorithm for the Bin Packing problem is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Prove or disprove:

(a) Next Fit is monotone.

(b) First Fit is monotone.

(4 points)

Exercise 5.2. Consider the Multiprocessor Scheduling Problem: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(a) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a 2-approximation algorithm.

(b) Is the analysis in (a) tight?

(c) Show that the modification of the greedy algorithm in which jobs are first sorted by $t(a)$ in non-increasing order and are then processed in that order is a $\frac{3}{2}$-approximation.

(2+1+3 points)

Deadline: Tuesday, May 14th, until 2:15 PM (before the lecture) on paper or per upload on eCampus. Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/appr_ss24_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.