Prof. Dr. Vera Traub
Luise Puhlmann

Exercise Set 3

Exercise 3.1. Consider the following local search algorithm for the Maximum Cut problem: Start with an arbitrary vertex set $S \subseteq V$. Iterate the following: If a single vertex can be added to S or can be removed from S such that $|\delta(S)|$ increases, do so. If no such vertex exists, terminate and return $\delta(S)$.

(a) Prove that this algorithm is a 2-approximation algorithm. (In particular, show that it runs in polynomial time.)

(b) Find an example that proves that the analysis is tight, even if we start with $S = \emptyset$.

(c) Does the algorithm always find an optimum solution for planar graphs or bipartite graphs?

(d) Give a linear time 2-approximation algorithm for the Maximum Cut problem in graphs with nonnegative edge weights.

(2+2+2+2 points)

Exercise 3.2. The k-Center Problem is defined as follows: given an undirected graph G, weights $c : E(G) \to \mathbb{R}_+$, and a number $k \in \mathbb{N}$ with $k \leq |V(G)|$, find a set $X \subseteq V(G)$ of cardinality k such that

$$\max_{v \in V(G)} \min_{x \in X} \operatorname{dist}(v, x)$$

is minimum. As usual we denote the optimum value by $\operatorname{OPT}(G, c, k)$.

(a) Let S be a maximal stable set in $(V(G), \{\{v, w\} \ : \ \operatorname{dist}(v, w) \leq 2R\})$. Show that then $\operatorname{OPT}(G, c, |S| - 1) > R$.

(b) Use (a) to describe a 2-factor approximation algorithm for the k-Center Problem.

(c) Prove that it is NP-hard to obtain an r-approximation for the k-Center Problem for any $r < 2$.

Hint: Use a reduction from the Vertex Cover Problem.

(7 points)

Exercise 3.3. Prove: If there is a 2-approximation algorithm for the maximum stable set problem, there is also a $(1+\epsilon)$-approximation algorithm for every constant $\epsilon > 0$.

(5 points)

Deadline: Tuesday, April 30th, until 2:15 PM (before the lecture) on paper or per upload on eCampus. Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/appr_ss24_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.