Exercise Set 1

Exercise 1.1. Formulate linear-time 2-approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

(a) Given an undirected, unweighted graph G, determine $v, w \in V(G)$ such that their distance is maximum.

(b) Given a directed graph G with non-negative edge weights, find an acyclic subgraph of maximum weight.

(c) Maximum-Satisfiability: Given an instance for Satisfiability, determine an assignment of truth values satisfying the maximum number of clauses.

(6 points)

Exercise 1.2. Given a directed cycle $C = (V, E)$ and a set of undirected edges $E_1 \subseteq \{\{v,w\} | v, w \in V, v \neq w\}$. We are looking for an orientation $E_1^{\leftrightarrow}$ of E_1 such that in the digraph $G' = (V, E \cup E_1^{\leftrightarrow})$,

$$\max_{e \in E} |\{C' \text{ directed cycle} | e \in E(C') \text{ with } |E(C') \cap E_1^{\leftrightarrow}| = 1\}|$$

is minimum. Give a linear time 2-approximation algorithm for that problem.

(4 points)

Exercise 1.3. Consider the following procedure for (unweighted) Minimum Vertex Cover: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(4 points)

Exercise 1.4. The LP relaxation of the Minimum Weight Vertex Cover Problem is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where $M \in \{0,1\}^{n \times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(i) Show that the above LP relaxation of the Minimum Weight Vertex Cover Problem always has a half-integral optimum solution.

(ii) Use this to obtain a 2-approximation algorithm. Is the analysis of the approximation ratio of this algorithm tight? (In other words, is the algorithm also a $(2-\varepsilon)$-approximation for some $\varepsilon > 0$?)

(4+2 points)

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Deadline: Tuesday, April 16th, until 2:15 PM (before the lecture) on paper or per upload on eCampus. Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/appr_ss24_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.