Exercise Set 12

Exercise 12.1. In this exercise, we use the setting of section 5.5.2 in the lecture notes. Let $c\colon E \to \mathbb{R}_{>0}$ depend only on direction and layer.

(a) Let $s = (x_s, y_s, z_s) \in V$, $y_v \in \mathbb{Z}$ and $v_i = (x_s, y_v, i) \in V$ for $i = 1, \dots, l$. Show that shortest paths from s to v_i for $i = 1, \dots, l$ can be computed in $\mathcal{O}(l)$ total time (so not just individually in linear time, but all l).

(b) Show that, without preprocessing time, one can compute $\operatorname{dist}_{G,c}(s, T)$ for any given $s \in V$ and given $T \subseteq V$ consisting of t rectangles in $\mathcal{O}(tl)$ time (you may use the first part as a black box).

(5+5 points)

Exercise 12.2. Let G be an undirected graph with resistances and capacitances $\operatorname{res}, \operatorname{cap}\colon E \to \mathbb{R}_{>0}$ on the edges. Let Y be a Steiner tree on the terminal set N rooted at the source pin p and sink pins $N \setminus \{p\}$. We interpret Y as an arborescence. Let $\pi\colon N \setminus \{p\} \to \mathbb{R}_{>0}$ be sink capacitances. We denote by $Y[p, q]$ the unique path from p to q in Y.

We define the Elmore delay from p to a sink $q \in N \setminus \{p\}$ in Y as

$$\operatorname{Elmore}_Y(p, q) := \sum_{e=(v,w)\in Y[p,q]} \operatorname{res}(e) \left(\frac{\operatorname{cap}(e)}{2} + \operatorname{downcap}(w) \right)$$

where $\operatorname{downcap}(q)$ is $\pi(q)$ if q is a sink pin and

$$\operatorname{downcap}(w) := \sum_{e=(w,x)\in\delta^+_Y(w)} (\operatorname{cap}(e) + \operatorname{downcap}(x))$$

otherwise.

Show that subdividing an edge $e = \{x, y\} \in E(G)$ into edges $e_1 = \{x, z\}$ and $e_2 = \{z, y\}$ using a new vertex z with $\operatorname{cap}(e_1) + \operatorname{cap}(e_2) = \operatorname{cap}(e)$, $\operatorname{res}(e_1) + \operatorname{res}(e_2) = \operatorname{res}(e)$ and $\frac{\operatorname{res}(e_1)}{\operatorname{cap}(e_1)} = \frac{\operatorname{res}(e_2)}{\operatorname{cap}(e_2)}$ does not change the Elmore delay.

(5 points)

Prof. Dr. Jens Vygen
Martin Drees, M. Sc.

Exercise 12.3. Let G be an undirected graph with weights, resistances and capacitances $w, \mathrm{res}, \mathrm{cap}\colon E \to \mathbb{R}_{>0}$ on the edges. Let $\alpha > 0$. For a path P from a to b in G let

$$\mathrm{cost}(P) := w(P) + \alpha\, \mathrm{Elmore}_P(a, b) ,$$

where P is interpreted as Steiner tree with two terminals rooted at a and $\pi(b) = 1$.

Let $s, t \in V(G)$. Assume that $\mathrm{cap}(e) = 1$ for all $e \in E(G)$.

Show that one can in polynomial time compute a path P from s to t with minimal cost.

Hint: Use a variation of Dijkstra's algorithm starting from t with labels $(v, \mathrm{downcap}(v))$ for vertices v.

(5 points)

Deadline: July 4, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss23/chipss23_ex.html`

In case of any questions feel free to contact me at drees@or.uni-bonn.de.