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Exercise Set 6

Exercise 6.1. Consider the *spreading LP* for $d = 2$:

$$\begin{aligned}
\min \quad & \sum_{e \in E(G)} w(e)\, l(e) \\
\text{s.t.} \quad & \sum_{y \in X} l(\{x,y\}) \geq \frac{1}{4}\left(|X|-1\right)^{3/2} && x \in X \subseteq V(G) \\
& l(\{x,y\}) + l(\{y,z\}) \geq l(\{x,z\}) && x,y,z \in V(G) \\
& l(\{x,y\}) \geq 1 && x,y \in V(G),\ x \neq y \\
& l(\{x,x\}) = 0 && x \in V(G)
\end{aligned}$$

Show that the optimum of the spreading LP is a lower bound for the cost of any 2-dimensional arrangement.

(5 points)

Exercise 6.2. Let $n \geq 4$ be a power of 2 and let $C_n = (V, E)$ be a cycle on n vertices. Let $l \colon V \times V \to \mathbb{R}_{\geq 0}$ be the metric given by the distances in C_n.

(a) Give an example of a 2-hierarchically well separated tree (T, r, c) such that V is the set of leaves of T and the induced tree metric l' satisfies

- $l'(x,y) \geq l(x,y)$ for all $x, y \in V$,
- $\sum_{\{x,y\} \in E} l'(x,y) \leq \mathcal{O}(\log n) \sum_{\{x,y\} \in E} l(x,y)$.

(b) Show that there exists a randomized polynomial time algorithm that computes a 2-hierarchically well separated tree (T, r, c) such that V is the set of leaves of T and the induced tree metric l' satisfies

- $l'(x,y) \geq l(x,y)$ for all $x, y \in V$,
- $\mathbb{E}[l'(x,y)] \leq \mathcal{O}(\log n) l(x,y)$ for all $x, y \in V$.

(c) Show that there exists no 2-hierarchically well separated tree (T, r, c) such that V is the set of leaves of T and the induced tree metric l' satisfies

- $l'(x,y) \geq l(x,y)$ for all $x, y \in V$,

- $l'(x, y) \leq \mathcal{O}(\log n) l(x, y)$ for all $x, y \in V$.

(2+2+3 points)

As May 18 is a public holiday, there will be no tutorial for this sheet. As a replacement, there will be individual meetings (in exercise groups) May 19 or May 22 where we discuss your submission, the first programming exercise or material from the lecture. You can select a slot for this meeting at

`https://terminplaner6.dfn.de/p/`
`205a73659f13c5a279af4552d529ad70-241093`

A digital submission by e-mail is recommended for this sheet.

Deadline: May 16, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss23/chipss23_ex.html`

In case of any questions feel free to contact me at drees@or.uni-bonn.de.