Exercise Set 5

Exercise 5.1. For a Boolean circuit C with inputs $1, \dots, n$ and arrival times $t_i \in \mathbb{N}$ ($i = 1, \dots, n$), its delay is defined as its depth after prepending a path with t_i circuits to input i ($i = 1, \dots, n$).

(a) Show that for n inputs with arrival times $t_i \in \mathbb{N}$ ($i = 1, \dots, n$) there are n-ary AND, OR or XOR circuits over B_2 with delay $d \in \mathbb{N}$ if and only if

$$\sum_{i=1}^{n} 2^{t_i - d} \leq 1.$$

(b) Provide an algorithm that finds such a circuit in $\mathcal{O}(n \log n)$ time.

(4 + 2 points)

Exercise 5.2. Provide a polynomial time algorithm for the Standard Placement Problem restricted to instances with only one circuit.

(4 points)

Exercise 5.3. Given a chip area A and a set $\mathcal{C}$ of circuits. A *movebound* for $C \in \mathcal{C}$ is a subset $A_C \subseteq A$ in which C must be placed entirely. Assume that the height and width of every circuit is 1 and that A and each movebound A_C ($C \in \mathcal{C}$) are axis-parallel rectangles with integral coordinates.

Describe an algorithm with running time polynomial in $|\mathcal{C}|$ that decides whether there is a feasible placement meeting all movebound constraints.

(5 points)

Exercise 5.4. The Gridded Placement Problem is an extension of the Standard Placement Problem with a grid $\Gamma = \Gamma_x \times \Gamma_y$ where $\Gamma_z := \{k \cdot \delta_z : k \in \mathbb{Z}\}$ with $\delta_z \in \mathbb{Z}$ for $z \in \{x, y\}$. In this variant, the lower left corner of each circuit is required to be in Γ.

Prove that the Gridded Placement Problem is NP-hard even if an optimum solution of the associated ungridded placement problem is known.

(5 points)

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Deadline: May 9, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss23/chipss23_ex.html`

In case of any questions feel free to contact me at drees@or.uni-bonn.de.