Exercise Set 3

Exercise 3.1. Let T be an instance of the Rectilinear Steiner Tree Problem and $r \in T$. For a rectilinear Steiner tree Y we denote by $f(Y)$ the maximum length of a path from r to any element of $T \setminus \{r\}$ in Y.

(a) Find an instance where no Steiner tree minimizes both length and f.

(b) Consider the problem of finding a shortest Steiner tree Y minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1 + 3 points)

Exercise 3.2. Let Y be a Steiner tree for terminal set T with $|T| \geq 2$ in which all leaves are terminals. Prove

$$\sum_{t \in T} \Big(|\delta_Y(t)| - 1 \Big) = k - 1$$

where k is the number of full components of Y.

(3 points)

Exercise 3.3. Let T be a finite set of terminals.

A topology for T is a tree X with $T \subseteq V(X)$, such that all Steiner points (elements of $V(X) \setminus T$) have degree at least 3. A topology is a standard topology if all Steiner points have degree exactly 3 in X and all elements of T are leaves.

We say that a Steiner tree Y has topology X if there is a map $\varphi \colon V(X) \to V(Y)$ with $\varphi(t) = t$ for all $t \in T$ such that $\{E(P_e) \colon e \in E(X)\}$ is a partition of $E(Y)$, where for $e = \{v, w\} \in E(X)$, P_e is the path from $\varphi(v)$ to $\varphi(w)$ in Y.

Modify the Dijkstra-Steiner Algorithm such that it finds an optimum Steiner tree with a fixed standard topology in $\mathcal{O}(k(m + n \log n))$ time, where $n = |V(G)|, m = |E(G)|$ and $k = |T|$.

(5 points)

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Deadline: April 25, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss23/chipss23_ex.html`

In case of any questions feel free to contact me at drees@or.uni-bonn.de.