Professor Dr. Vera Traub
Antonia Ellerbrock

Exercise Set 9

Exercise 9.1. Let (G, c, T) be an instance of Steiner Tree. Fix $k \in \mathbb{N}$. Let $\mathcal{C}^k$ be the set of pairs (t, R) for all $t \in R \subseteq T$ with $|R| \leq k$ (called *directed components*). For $C = (t, R) \in \mathcal{C}^k$, let $c(C)$ denote the minimum cost of a Steiner tree for R in (G, c). Fix an arbitrary terminal $r \in T$ as root. For $r \in U \subset T$, we denote by $\delta^+_{\mathcal{C}^k}(U)$ the set of all $(t, R) \in \mathcal{C}^k$ with $t \in U$ and $R \setminus U \neq \emptyset$. The so-called *directed component LP* then reads as follows:

$$
\begin{aligned}
\min \quad & \sum_{C \in \mathcal{C}^k} c(C) x_C & \\
s.t. \quad & \sum_{C \in \delta^+_{\mathcal{C}^k}(U)} x_C \geq 1 & (r \in U \subset T) \\
& x_C \geq 0 & (C \in \mathcal{C}^k)
\end{aligned}
$$

Prove that for any fixed $k \in \mathbb{N}$, the directed component LP for the problem of finding a k-restricted Steiner tree can be solved in polynomial time.

(5 points)

Exercise 9.2. Show that the clean-up step of the Primal-Dual Algorithm For Steiner Forest (the step that (possibly) removes edges from F) is crucial: Without this step, the algorithm does not even achieve any finite approximation ratio.

(5 points)

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo (link on website, late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, June 13th, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.