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Exercise Set 7

Exercise 7.1. Let $(U, \mathcal{S}, c)$ be an instance of MINIMUM WEIGHT SET COVER and $p := \max_{S \in \mathcal{S}} |S|$. Consider the following algorithm:

1. For $u \in U$, define $c_u := \min\{c(S) : u \in S \in \mathcal{S}\}$.

2. Let $\mathcal{R} := \emptyset$ and let $W := U$ be the set of elements uncovered by $\mathcal{R}$.

3. While $\mathcal{R}$ is not a feasible solution, do the following:

- Choose a set $S \in \mathcal{S}$ that minimizes $\frac{c(S)}{\sum_{u \in (S \cap W)} c_u}$.
- Add S to $\mathcal{R}$.
- Replace W by $(W \setminus S)$.

4. Return $\mathcal{R}$.

(a) Prove that in every iteration of the above algorithm, we have

$$\frac{c(S)}{\sum_{u \in (S \cap W)} c_u} \leq \min\left\{1 \ , \ \frac{\text{OPT}}{\sum_{u \in W} c_u}\right\}$$

where OPT denotes the value of an optimum solution.

(b) Prove that the above algorithm is a $(1 + \ln(p))$-approximation for MINIMUM WEIGHT SET COVER.

(2+3 points)

Exercise 7.2. Show that for any terminal spanning tree (T, S) and any k-component X, the set

$$\mathcal{B} := \{D \subseteq S : (S \setminus D) \cup E(X) \text{ is the edge set of a connector for } T\}$$

is the set of independent sets of a matroid.

(5 points)

Lecture Course Evaluation: In the lecture on Tuesday, May 23$^{\text{rd}}$, you will have the chance to give constructive feedback on the lectures and tutorials via an anonymous online survey. Please bring an internet enabled device, WiFi access will be provided.

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo to

`https://uni-bonn.sciebo.de/s/omVU1VMioEQwDa0`

(late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, May 23$^{\text{rd}}$, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.