Exercise Set 6

Exercise 6.1. Consider the following algorithm for the STEINER TREE PROBLEM with 3 terminals v_1, v_2 and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

(i) $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log(|V|))$ time and works correctly.

(4 points)

Exercise 6.2. Let $\varepsilon > 0$. Show that a k-approximation algorithm for the STEINER TREE PROBLEM with unit edge weights $c \equiv 1$ can be used to obtain a $(k + \varepsilon)$-approximation algorithm for the weighted STEINER TREE PROBLEM with edge weights $c : E(G) \to \mathbb{R}_+$.

(4 points)

Exercise 6.3. Consider the DIRECTED STEINER TREE PROBLEM: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.

Show that a k-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a k-approximation algorithm for MINIMUM WEIGHT SET COVER.

(4 points)

Exercise 6.4. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender. Prove:

(a) The restriction of this problem to instances with $S \cup R = V$ is in P.

(b) This problem is NP-hard. Give a 2-approximation algorithm.

(2+2 points)

Exercise 6.5. Prove that the 4-Steiner ratio ρ_4 is $\frac{3}{2}$.

(4 points)

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo to

`https://uni-bonn.sciebo.de/s/omVU1VMioEQwDa0`

(late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, May 16th, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.