Exercise Set 5

Exercise 5.1. Give an algorithm for Bin Packing restricted to instances with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

Hint: Use dynamic programming.

(4 points)

Exercise 5.2. For an instance of Bin Packing, let $(s_1, \dots, s_m)$ denote the different item sizes and $(b_1, \dots, b_m)$ their multiplicities. Denote by

$$\{T_1, \dots, T_N\} = \left\{ (k_1, \dots, k_m) \in \mathbb{Z}_+^m : \sum_{i=1}^m k_i s_i \leq 1 \right\}$$

all possible configurations for a single bin, where $T_j = (t_{j1}, \dots, t_{jm})$ for $j = 1, \dots, N$. Consider the following LP:

$$\begin{aligned} \min \quad & \sum_{j=1}^N x_j \\ s.t. \quad & \sum_{j=1}^N t_{ji} x_j \geq b_i && (i = 1, \dots, m) \\ & x_j \geq 0 && (j = 1, \dots, N) \end{aligned}$$

Let LP denote the optimum value of this LP and let OPT denote the value of an optimum integral solution (i.e. an optimum solution to the Bin Packing problem).

Show that there exists an instance of Bin Packing with $\lceil \text{LP} \rceil < \text{OPT}$.

(4 points)

Exercise 5.3. Show that if all item sizes are of the form $a_i = k \cdot 2^{-b_i}$ for some $b_i \in \mathbb{N}$ $(i = 1, \dots, n)$ and some fixed $k \in \mathbb{N}$, then the First Fit Decreasing algorithm always finds an optimum solution.

(4 points)

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Exercise 5.4.

(a) Prove that for any fixed $\varepsilon > 0$ there exists a polynomial-time algorithm which finds a packing using the optimum number of bins but possibly violating the capacity constraints by ε for any instance $I = (a_1, \ldots, a_n)$ of Bin Packing, i.e. an $f : \{1, \ldots, n\} \to \{1, \ldots, \mathrm{OPT}(I)\}$ with $\sum_{f(i)=j} a_i \leq 1 + \varepsilon$ for all $j \in \{1, \ldots, \mathrm{OPT}(I)\}$.

Hint: Use Exercise 5.1.

(b) Use (a) to show that the Multiprocessor Scheduling Problem (see Exercise 4.4) has an approximation scheme.

(4+4 points)

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo to

`https://uni-bonn.sciebo.de/s/omVU1VMioEQwDa0`

(late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, May 9$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.