Professor Dr. Vera Traub
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Exercise Set 4

Exercise 4.1. Show that the following variant of KNAPSACK is NP-hard:

$$\max\left\{\sum_{i=1}^{n} c_i x_i \,:\, \sum_{i=1}^{n} w_i x_i \leq W,\, x_i \in \mathbb{Z}_{\geq 0}\, \forall\, 1 \leq i \leq n\right\}$$

(We allow to use an item several times.)

You may use that KNAPSACK is NP-hard.

(5 points)

Exercise 4.2. Consider the following variant of PARTITION: Given $a_1, \ldots, a_n \in \mathbb{Z}_+$, find a set $S \subseteq \{1, \ldots, n\}$ such that

$$\max\left\{\sum_{i \in S} a_i \;,\; \sum_{i \in \{1,\ldots,n\} \setminus S} a_i\right\}$$

is minimum.

Show that this problem has a fully polynomial approximation scheme.

(3 points)

Exercise 4.3. Let $A = (a_i)_{1 \leq i \leq p}$ and $B = (b_j)_{1 \leq j \leq q}$ be two inputs of BIN PACKING. We write $A \subseteq B$ if there are indices $1 \leq k_1 < k_2 < \cdots < k_p \leq q$ with $a_i \leq b_{k_i}$ for $1 \leq i \leq p$. An algorithm for BIN PACKING is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Prove or disprove:

(a) NEXT FIT is monotone.

(b) FIRST FIT is monotone.

(c) FIRST FIT DECREASING is monotone.

(2+2+2 points)

Exercise 4.4. Consider the Multiprocessor Scheduling Problem: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(a) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a 2-approximation algorithm.

(b) Is the anaylsis in (a) tight?

(c) Show that the modification of the greedy algorithm in which jobs are first sorted by $t(a)$ in non-increasing order and are then processed in that order is a $\frac{3}{2}$-approximation.

(2+1+3 points)

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo to

`https://uni-bonn.sciebo.de/s/omVU1VMioEQwDa0`

(late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, May 2nd, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.