Professor Dr. Vera Traub
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Exercise Set 2

Exercise 2.1. Prove: If there is a 2-approximation algorithm for the maximum stable set problem, there is also a $(1+\epsilon)$-approximation algorithm for every $\epsilon > 0$.
(4 points)

Exercise 2.2. For $k \in \mathbb{N}$ consider the following problem:

Instance: A set U and a set $\mathcal{S}$ of subsets of U with $|S| \leq k$ for all $S \in \mathcal{S}$, weights $w : U \to \mathbb{R}_{\geq 0}$.

Task: Find $T \subseteq U$ such that $T \cap S \neq \emptyset$ for each $S \in \mathcal{S}$ and $\sum_{t \in T} w(t)$ minimum.

(i) Show that this problem is NP-hard for $k \geq 2$.

(ii) Give a polynomial time k-factor approximation algorithm.

(iii) Give a linear time k-factor approximation algorithm for the special case that $w(t) = 1$ for $t \in U$.

(1+2+2 points)

Exercise 2.3. Consider the standard IP formulation of the Minimim Weight Set Cover Problem, and its LP-relaxation

$$\min \left\{ cx \ : \ \sum_{S \in \mathcal{S} : e \in S} x_S \geq 1 \text{ for all } e \in U,\ x_S \geq 0 \text{ for all } S \in \mathcal{S} \right\}.$$

Consider the algorithm that picks all sets associated with non-zero values in an optimum solution to the LP-relaxation. Show that this algorithm achieves an approximation guarantee of p if each element $e \in U$ is contained in at most p sets.
(3 points)

Exercise 2.4. Consider the following variant of Set Cover:

Instance:	A set U, a set $\mathcal{S}$ of subsets of U with $\bigcup_{S \in \mathcal{S}} S = U$, an integer $k \in \mathbb{N}$.
Task:	Find k sets $S_1, \ldots, S_k \in \mathcal{S}$ such that $\left\| \bigcup_{j=1}^{k} S_j \right\|$ is maximum.

Show that iteratively picking the set that maximizes the number of not yet covered elements is a $(\frac{e}{e-1})$-approximation.

(4 points)

Exercise 2.5. The restriction of Satisfiability to instances where each clause consists of exactly two literals is called 2-Satisfiability.

Prove that 2-Satisfiability is in P.

(4 points)

Submission: You can submit your solutions in groups of 2 people, either on paper in the lecture or via upload on Sciebo to

`https://uni-bonn.sciebo.de/s/omVU1VMioEQwDa0`

(late submissions after 2.15 pm will not be considered).

Deadline: Tuesday, April 18^{th}, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss23/ss23.html`

In case of any questions feel free to contact me at ellerbrock@or.uni-bonn.de.