The Simplex Algorithm

Algorithm 1: Simplex Algorithm

Input: $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$

Output: $\tilde{x} \in \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$ maximizing $c^t x$ or the message that $\max\{c^t x \mid Ax = b, x \geq 0\}$ is unbounded or infeasible

1 Compute a feasible basis B;

2 If no such basis exists, stop with the message "INFEASIBLE";

3 Set $N = \{1, \ldots, n\} \setminus B$ and compute the feasible basic solution x for B;

4 Compute the simplex tableau $\begin{array}{rcrcl} x_B & = & p & + & Qx_N \\ z & = & z_0 & + & r^t x_N \end{array}$ for B;

5 **if** $r \leq 0$ **then**
  return $\tilde{x} = x$;

6 Choose $\alpha \in N$ with $r_\alpha > 0$;

7 **if** $q_{i\alpha} \geq 0$ **for all** $i \in B$ **then**
  return "UNBOUNDED";

8 Choose $\beta \in B$ with $q_{\beta\alpha} < 0$ and $\frac{p_\beta}{q_{\beta\alpha}} = \max\{\frac{p_i}{q_{i\alpha}} \mid q_{i\alpha} < 0, i \in B\}$;

9 Set $B = (B \setminus \{\beta\}) \cup \{\alpha\}$;

10 GOTO line 3;

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Algorithm 2: Simplex Algorithm

Input: $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$

Output: $\tilde{x} \in \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$ maximizing $c^t x$ or the message that $\max\{c^t x \mid Ax = b, x \geq 0\}$ is unbounded or infeasible

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  return "UNBOUNDED";

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Algorithm 3: Simplex Algorithm

Input: $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$

Output: $\tilde{x} \in \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$ maximizing $c^t x$ or the message that $\max\{c^t x \mid Ax = b, x \geq 0\}$ is unbounded or infeasible

1 Compute a feasible basis B;

2 If no such basis exists, stop with the message "INFEASIBLE";

3 Set $N = \{1, \ldots, n\} \setminus B$ and compute the feasible basic solution x for B;

4 Compute the simplex tableau $\begin{array}{rcl} x_B & = & p + Qx_N \\ z & = & z_0 + r^t x_N \end{array}$ for B;

5 **if** $r \leq 0$ **then**
  return $\tilde{x} = x$;

6 Choose $\alpha \in N$ with $r_\alpha > 0$;

7 **if** $q_{i\alpha} \geq 0$ **for all** $i \in B$ **then**
  return "UNBOUNDED";

8 Choose $\beta \in B$ with $q_{\beta\alpha} < 0$ and $\frac{p_\beta}{q_{\beta\alpha}} = \max\{\frac{p_i}{q_{i\alpha}} \mid q_{i\alpha} < 0, i \in B\}$;

9 Set $B = (B \setminus \{\beta\}) \cup \{\alpha\}$;

10 GOTO line 3;