## Exercise Set 8

**Exercise 8.1.** Consider following specialized legalization problem. A feasible placement consists of integral placement coordinates  $x, y : \mathcal{C} \to \mathbb{Z}^2$  (such that all circuits are located within the chip image), and all circuits have unit height and width. In addition, there is an (infeasible) input placement  $\tilde{x}, \tilde{y} : \mathcal{C} \to \mathbb{R}^2$ .

- Find a polynomial-time algorithm that finds a legal placement minimizing the linear or quadratic movement.
- Find a linear time algorithm that finds a legal placement minimizing the quadratic movement if the instance consists of a single row. Assume that the input is sorted.

(2+2 points)

**Exercise 8.2.** Consider the following variant of the SINGLE ROW PLACE-MENT WITH FIXED ORDERING problem, in which we minimize the bounding box net length:

**Input:** A set  $C = \{C_1, \ldots, C_n\}$  of circuits, widths  $w(C_i) \in \mathbb{R}_+$ , an interval  $[0, w(\Box)]$ , s.t.  $\sum_{i=1}^n w(C_i) \leq w(\Box)$ . A netlist  $(C, P, \gamma, \mathcal{N})$ where the offset of a pin  $p \in P$  satisfies  $x(p) \in [0, w(\gamma(p))]$ . Weights  $\alpha : \mathcal{N} \to \mathbb{R}_+$ .

**Task:** Find a feasible placement given by a function  $x : \mathcal{C} \to \mathbb{R}$ s.t.  $0 \leq x(C_1), x(C_i) + w(C_i) \leq x(C_{i+1})$  for  $i = 1, \ldots, n-1$  and  $x(C_n) + w(C_n) \leq w(\Box)$ , that minimizes

$$\sum_{N \in \mathcal{N}} \alpha(N) \cdot \mathrm{BB}(N).$$

Here, BB(N) denotes the bounding box net length.

Show that there exist  $f_i : [0, w(\Box)] \to \mathbb{R}$ , i = 1, ..., n, piecewise linear, continuous and convex, such that we can solve this problem by means of the SINGLE ROW ALGORITHM.

(5 points)

**Exercise 8.3.** Consider an instance of the MULTISECTION PROBLEM with k regions and a feasible fractional assignment. Prove that there is an integral partition which violates capacity constraints by at most

$$\frac{k-1}{k} \max\left\{ \operatorname{size}(C) : C \in \mathcal{C} \right\}.$$

(5 points)

**Exercise 8.4.** Consider the ESCAPE ROUTING PROBLEM: We are given a complete 2-dimensional grid graph G = (V, E) (i.e.  $V = \{0, \ldots, k-1\} \times \{0, \ldots, k-1\}$  and  $E = \{\{v, w\} \mid v, w \in V, ||v - w|| = 1\}$ ) and a set  $P = \{p_1, \ldots, p_m\} \subseteq V$ . The task is to compute vertex-disjoint paths  $\{q_1, \ldots, q_m\}$  s.t. each  $q_i$  connects  $p_i$  with a point on the border  $B = \{(x, y) \in V \mid \{x, y\} \cap \{0, k-1\} \neq \emptyset\}$ .

Find a polynomial-time algorithm for the ESCAPE ROUTING PROBLEM or prove that the problem is NP-hard.

(5 points)

**Deadline:** June 14, before the lecture. The websites for lecture and exercises can be found at:

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http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html
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In case of any questions feel free to contact me at blankenburg@or.unibonn.de.