## Exercise Set 8

Exercise 8.1. Consider following specialized legalization problem. A feasible placement consists of integral placement coordinates $x, y: \mathcal{C} \rightarrow \mathbb{Z}^{2}$ (such that all circuits are located within the chip image), and all circuits have unit height and width. In addition, there is an (infeasible) input placement $\tilde{x}, \tilde{y}: \mathcal{C} \rightarrow \mathbb{R}^{2}$.

- Find a polynomial-time algorithm that finds a legal placement minimizing the linear or quadratic movement.
- Find a linear time algorithm that finds a legal placement minimizing the quadratic movement if the instance consists of a single row. Assume that the input is sorted.

$$
\text { ( } 2+2 \text { points })
$$

Exercise 8.2. Consider the following variant of the Single Row Placement With Fixed Ordering problem, in which we minimize the bounding box net length:

Input: A set $\mathcal{C}=\left\{C_{1}, \ldots, C_{n}\right\}$ of circuits, widths $w\left(C_{i}\right) \in \mathbb{R}_{+}$, an interval $[0, w(\square)]$, s.t. $\sum_{i=1}^{n} w\left(C_{i}\right) \leq w(\square)$. A netlist $(\mathcal{C}, P, \gamma, \mathcal{N})$ where the offset of a pin $p \in P$ satisfies $x(p) \in[0, w(\gamma(p))]$. Weights $\alpha: \mathcal{N} \rightarrow \mathbb{R}_{+}$.

Task: Find a feasible placement given by a function $x: \mathcal{C} \rightarrow \mathbb{R}$ s.t. $0 \leq x\left(C_{1}\right), x\left(C_{i}\right)+w\left(C_{i}\right) \leq x\left(C_{i+1}\right)$ for $i=1, \ldots, n-1$ and $x\left(C_{n}\right)+w\left(C_{n}\right) \leq w(\square)$, that minimizes

$$
\sum_{N \in \mathcal{N}} \alpha(N) \cdot \operatorname{BB}(N)
$$

Here, $\mathrm{BB}(N)$ denotes the bounding box net length.
Show that there exist $f_{i}:[0, w(\square)] \rightarrow \mathbb{R}, i=1, \ldots, n$, piecewise linear, continuous and convex, such that we can solve this problem by means of the Single Row Algorithm.

Exercise 8.3. Consider an instance of the Multisection Problem with $k$ regions and a feasible fractional assignment. Prove that there is an integral partition which violates capacity constraints by at most

$$
\frac{k-1}{k} \max \{\operatorname{size}(C): C \in \mathcal{C}\}
$$

Exercise 8.4. Consider the Escape Routing Problem: We are given a complete 2-dimensional grid graph $G=(V, E)$ (i.e. $V=\{0, \ldots, k-1\} \times$ $\{0, \ldots, k-1\}$ and $E=\{\{v, w\} \mid v, w \in V,\|v-w\|=1\})$ and a set $P=$ $\left\{p_{1}, \ldots, p_{m}\right\} \subseteq V$. The task is to compute vertex-disjoint paths $\left\{q_{1}, \ldots, q_{m}\right\}$ s.t. each $q_{i}$ connects $p_{i}$ with a point on the border $B=\{(x, y) \in V \mid\{x, y\} \cap$ $\{0, k-1\} \neq \emptyset\}$.

Find a polynomial-time algorithm for the Escape Routing Problem or prove that the problem is NP-hard.
(5 points)

Deadline: June 14, before the lecture. The websites for lecture and exercises can be found at:
http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html
In case of any questions feel free to contact me at blankenburg@or.unibonn.de.

