Prof. Dr. Stephan Held
Daniel Blankenburg, M. Sc.

Exercise Set 5

Exercise 5.1. Consider the following algorithm to compute a rectilinear Steiner tree Y for a set T of points in the plane $\mathbb{R}^2$.

Choose $t \in T$ arbitrarily;
$Y := (\{t\}, \emptyset), S := T \setminus \{t\}$
while $S \neq \emptyset$ **do**
    Choose $s \in S$ with minimum $dist(s, Y)$
    if $E(Y) = \emptyset$ **then**
        $Y := (\{t, s\}, \{\{t, s\}\})$
    else
        Let $\{u, w\} \in E(Y)$ be an edge which minimizes $dist(s, SP(u, w))$
        $v := \arg\min\{dist(s, v) \mid v \in SP(u, w)\}$
        $Y := (V(Y) \cup \{v\} \cup \{s\}, E(Y) \setminus \{\{u, w\}\} \cup \{\{u, v\}, \{v, w\}, \{v, s\}\})$
    end if
    $S := S \setminus \{s\}$
end while

In this notation $SP(u, w) \subset \mathbb{R}^2$ is the area covered by shortest paths between u and w, and $dist(s, Y)$ is the minimum distance between s and the shortest path area $SP(u, w)$ of an edge $\{u, w\} \in E(Y)$.

Show that the algorithm is a $\frac{3}{2}$-approximation algorithm for the MINIMUM STEINER TREE PROBLEM.

(5 points)

Exercise 5.2. Let T be an instance of the Rectilinear Steiner Tree Problem and $r \in T$. For a rectilinear Steiner tree Y we denote by $f(Y)$ the maximum length of a path from r to any element of $T \setminus \{r\}$ in Y.

(a) Find an instance where no Steiner tree minimizes both length and f.

(b) Consider the problem of finding a shortest Steiner tree Y minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1 + 4 points)

Exercise 5.3. Let Y be a Steiner tree for terminal set T with $|T| \geq 2$ in which all leaves are terminals. Prove

$$\sum_{t \in T} \Big(|\delta_Y(t)| - 1 \Big) = k - 1$$

where k is the number of full components of Y.

(5 points)

Exercise 5.4. Given $u, v \in \mathbb{R}^2$, let

$$\mathcal{L}(u,v) := \{x \in \mathbb{R}^2 : \max\{||x-u||_1, ||x-v||_1\} < ||v-u||_1\}.$$

Let Y be a canonical tree that is full. Prove that any pair of distinct edges $(u,v), (x,y) \in E(Y)$ satisfies $\mathcal{L}(u,v) \cap \mathcal{L}(x,y) = \emptyset$.

Remark: This criterion can be used to prune prospective full components.

(5 points)

Deadline: May 12, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html`

In case of any questions feel free to contact me at blankenburg@or.uni-bonn.de.