

Exercise Set 2

Exercise 2.1. A Boolean function $f \in B_n$ *depends essentially* on all its variables if for every $1 \leq i \leq n$ the subfunctions $f|_{x_i=0}$ and $f|_{x_i=1}$ are different.

Let $f \in B_n$ be a function that essentially depends on all its variables. Show:

(a) $S_{B_2}(f) \geq n - 1$,

(b) $D_{B_2}(f) \geq \lceil \log_2 n \rceil$.

(5 points)

Exercise 2.2. Define a class of functions $(f_n)_{n \in \mathbb{N}}$ such that $f_n \in B_n$ and their SOP representations have size $\Omega(2^n)$.

(5 points)

Exercise 2.3. In the following we use the notation

$$S_{B_2}(B_n) := \max\{S_{B_2}(f) : f \in B_n\}$$

for $n \in \mathbb{N}$. Further, for $f \in B_n$, let $f_0 := f|_{x_n=0} \in B_{n-1}$ and $f_1 = f|_{x_n=1} \in B_{n-1}$. One can write f as a *decoding circuit* in the following way:

$$f = (\bar{x}_n \wedge f_0) \vee (x_n \wedge f_1). \quad (1)$$

By applying this representation recursively, it follows that $S_{B_2}(B_n) \leq 2^n - 3$. The goal of this exercise is to improve this bound.

(a) Let $S_{B_2}^*(B_k)$ denote the circuit complexity of computing all functions in B_k (such a circuit has 2^{2^k} outputs). Use (1) to show that

$$S_{B_2}^*(B_k) \in \mathcal{O}(2^{2^k}).$$

(b) Use (a) to show that $S_{B_2}(B_n) \in \mathcal{O}(2^n/n)$.

(2+3 points)

Exercise 2.4. Let $n \in \mathbb{N}$. Construct a prefix graph that has

- $\{o\}$ -depth at most $\log n + \log \log n + \mathcal{O}(1)$,
- $\{o\}$ -size at most $3n + \mathcal{O}(1)$,
- and a maximum fanout of two.

You may use repeaters in your construction.

(5 points)

Deadline: April 21, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html

In case of any questions feel free to contact me at blankenburg@or.uni-bonn.de.

Announcement by the gender equality committee: We invite all female, intersexual, non-binary, trans* and agender bachelor and master students to the in-person event "Tea Time With Women in Mathematics", which will take place on April 29th from 4pm (s.t.) to 6pm in the Zeichensaal, Wegelerstr. 10. You will get the chance to talk to other participants about your experiences during your studies, your plans for the future, and everything else that comes to your mind, whilst enjoying a nice cup of tea and some cookies.