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Exercise Set 12

Exercise 12.1. Describe a polynomial-time algorithm which optimally solves any instance of the TRAVELING SALESMAN PROBLEM that is the metric closure of a weighted tree.

(4 points)

Exercise 12.2. Let c_0 be the value of an optimal solution of an instance of the METRIC TSP and c_1 the cost of a second-shortest tour (note that this tour might have the same cost as the first one). Show that

$$\frac{c_1 - c_0}{c_0} \leq \frac{2}{n}.$$

(4 points)

Exercise 12.3. Consider the following algorithm for the SYMMETRIC TRAVELING SALESMAN PROBLEM with triangle inequality:

Start with an arbitrary vertex $u \in V(K_n)$. Find a shortest edge $e = \{u, v\} \in E(K_n)$ connecting u to another vertex v. This yields a subtour $T = (u, v, u)$. Let $U := V(K_n) \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.

(ii) Add w to T between two neighbouring nodes $i, j \in T$ (by deleting the edge $\{i, j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $c(i, w) + c(w, j) - c(i, j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation.

(4 points)

Exercise 12.4. Show that the following problem is NP-complete: Given a graph G and a Hamiltonian cycle C in G, is there a Hamiltonian cycle $C' \neq C$?

(4 points)

Deadline: Tuesday, July 5$^{\text{th}}$, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.