# Exercise Set 10

**Exercise 10.1.** Recall the LP $k$-DCR from Exercise 9.4. Consider the LP $n$-DCR where we allow directed full components of arbitrary size. Denote this LP by DCR. DCR has an exponential number of variables and constraints.

Show that, nevertheless, for any fixed $\epsilon > 0$, we can compute a $(1+\epsilon)$-approximate solution to DCR in polynomial time.

*Hint: Solve $k$-DCR for a convenient choice of $k$.*

(6 points)

**Exercise 10.2.** Recall the LPs DCR from Exercise 10.1 and BCR from Exercise 9.1. Denote by $\mathrm{OPT}_{\mathrm{DCR}}$ respectively $\mathrm{OPT}_{\mathrm{BCR}}$ the value of an optimum solution to DCR resp. BCR for a given input instance. Show:

(a) $\mathrm{OPT}_{\mathrm{DCR}} \geq \mathrm{OPT}_{\mathrm{BCR}}$ and

(b) there are examples where $\mathrm{OPT}_{\mathrm{DCR}} > \mathrm{OPT}_{\mathrm{BCR}}$.

(2+2 points)

**Exercise 10.3.** Consider an instance $G = (V, E)$ of the Steiner Tree Problem with terminal set $R$ and edge length $c : E \to \mathbb{R}_+$. Denote the full components of an optimum $k$-Steiner tree $\mathrm{SMT}_k(R)$ with $T_1^*, \ldots, T_l^*$.

(i) Suppose that $V \setminus R$ forms a stable set. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_l^*)) .$$

(ii) Suppose that all shortest paths between any two vertices in $G$ have length 1 or 2. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_l^*)) .$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

(2+2+2 points)

**Deadline:** June 21st, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.