Prof. Dr. Stefan Hougardy
Luise Puhlmann

Exercise Set 8

Exercise 8.1. Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.

(Note: You may use that the Voronoi regions are computed with Dijkstra's algorithm.)
(4 points)

Exercise 8.2. Consider the following greedy algorithm for the Graph Steiner Tree Problem: Given a graph $G = (V, E)$ with terminal set $R \subseteq V$ **such that** $V \setminus R$ **forms a stable set** (such instances are called quasi-bipartite) and edge lenghts $c\colon E \to \mathbb{R}_{\geq 0}$, we set R' initially to R and compute a minimum spanning tree $T = \mathrm{MST}(R')$ in the terminal distance graph $G_D(R')$. While there is some $v \in V \setminus R'$ with $c(\mathrm{MST}(R' \cup \{v\})) < c(\mathrm{MST}(R'))$, add v to R' and remove any non-terminals of degree ≤ 2 in $\mathrm{MST}(R')$ from R'. Return $\mathrm{MST}(R')$.

(a) Show that this algorithm is then a $\frac{3}{2}$-approximation algorithm.

(b) Show that this algorithm is no ρ-approximation for any $\rho < \frac{3}{2}$.

(4+4 points)

Exercise 8.3. Let T be a Steiner tree for a terminal set R. Assume that T is a binary tree and its leaves are exactly R. Let $r \leq |V(T) \setminus R|$ be given in the input. We would like to turn exactly r Steiner nodes into terminals such that we obtain a k-Steiner tree with k minimum. Give a polynomial time algorithm that solves this problem optimally.
(4 points)

Deadline: Tuesday, May 31st, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.