

Exercise Set 7

Exercise 7.1. Consider the restriction \mathcal{P} of the unweighted VERTEX COVER PROBLEM to graphs where the maximum degree of every vertex is bounded by a constant B .

Let $\epsilon > 0$. Show: If there exists a polynomial time approximation algorithm for the STEINER TREE PROBLEM with performance ratio $1 + \epsilon$, then there exists a polynomial time approximation algorithm for problem \mathcal{P} with performance ratio $1 + (B + 1)\epsilon$.

(4 points)

Exercise 7.2. Give an $\mathcal{O}(n^3t^2)$ algorithm for the STEINER TREE PROBLEM in planar graphs with all terminals lying on the outer face, where n is the number of vertices and t the number of terminals.

(Hint: Modify the Dreyfus-Wagner algorithm.)

(4 points)

Exercise 7.3. Consider the following algorithm for the STEINER TREE PROBLEM with 3 terminals v_1, v_2 and v_3 : Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P . Then find a vertex z minimizing $\sum_{i=1}^3 \text{dist}(v_i, z)$ under the conditions

- (i) $\text{dist}(v_i, z) \leq \text{dist}(v_1, v_2)$ for $i \in \{1, 2\}$ and
- (ii) $\text{dist}(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log(|V|))$ time and works correctly.

(4 points)

Exercise 7.4. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets (“senders” and “receivers”). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

- (a) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P .
- (b) Prove that this problem is NP-hard and give a 2-factor approximation algorithm.

(2+2 points)

Deadline: Tuesday, May 24th, until 2:15 PM (before the lecture) via eCampus. \LaTeX -submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.