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# Exercise Set 4

**Exercise 4.1.** Consider the following procedure for (unweighted) Minimum Vertex Cover: Given a graph $G$, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(3 points)

**Exercise 4.2.** Consider the Minimum Weight Vertex Cover Problem, and recall its LP relaxation, i.e. $\min\{cx : M^T x \geq 1, x \geq 0\}$, where $M \in \{0,1\}^{n \times m}$ is the incidence matrix of an undirected graph $G = (V, E)$ and $c \in \mathbb{R}_+^{V(G)}$. Assume that you are given a coloring $\varphi : V \to \{1, \ldots, k\}$ of $G$. Derive a $(2 - 2/k)$-approximation algorithm from Exercise 2.2.

(3 points)

**Exercise 4.3.** Let $G$ be a $k$-colorable graph with $n$ vertices, where $k$ is a constant. We define $x_k := n^{1-\frac{1}{k-1}}$ and for $2 \leq l < k$, $x_l := x_{l+1}^{1-\frac{1}{l-1}}$. For simplicity, we assume that $n$ is chosen such that $x_l$ is a natural number for $l \in \{2, \ldots, k\}$.

Prove that there exists a polynomial time algorithm that colors $G$ with $kx_k$ colors.

(5 points)

**Exercise 4.4.** An instance of Max-Sat is called $k$-satisfiable if any $k$ of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

*Hint:* Some variables occur in one-element clauses (w.l.o.g. all one-element clauses are positive), set them *true* with probability $a$ (for some constant $a \in [0,1]$), and set the other variables *true* with probability $\frac{1}{2}$. Choose $a$ appropriately and derandomize this algorithm.

(5 points)

**Deadline:** Tuesday, May 3rd, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.