Exercise Set 3

Exercise 3.1. Consider the following variant of the k-CENTER problem:

Instance: A complete graph $G = (V, E)$, metric edge weights $d : E(G) \to \mathbb{R}$, a partition $V = C \dot{\cup} S$, an integer $k \in \mathbb{N}$.

Output: A set $X \subseteq S$ with $|X| \leq k$ that minimizes

$$\max_{c \in C} \left\{ \min_{s \in X} \{d(c, s)\} \right\}.$$

(i) Show that this problem does not admit a $(3 - \varepsilon)$-approximation for any $\varepsilon > 0$ unless P=NP.

(ii) Give a 3-approximation algorithm.

(4+4 points)

Exercise 3.2. Formulate linear-time 2-factor approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

(a) Given an undirected, unweighted graph G, determine $v, w \in V(G)$ such that their distance is maximum.

(b) Given a directed graph G with non-negative edge weights, find an acyclic subgraph of maximum weight.

(c) MAXIMUM-SATISFIABILITY: Given an instance for SATISFIABILITY, determine an assignment of truth values satisfying the maximum number of clauses.

(2+2+2 points)

Exercise 3.3. Consider the DIRECTED STEINER TREE PROBLEM: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.

Show that a k-approximation algorithm for the DIRECTED STEINER TREE PROBLEM can be used to obtain a k-approximation algorithm for MINIMUM WEIGHT SET COVER.

(2 points)

Deadline: Tuesday, April 26th, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.