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Exercise Set 2

Exercise 2.1. For $k \in \mathbb{N}$ consider the following problem:

Instance: A set U and a set $\mathcal{S}$ of subsets of U with $|S| \leq k$ for all $S \in \mathcal{S}$, weights $w : U \to \mathbb{R}_{\geq 0}$.

Task: Find $T \subseteq U$ such that $T \cap S \neq \emptyset$ for each $S \in \mathcal{S}$ and $\sum_{t \in T} w(t)$ minimum.

(i) Show that this problem is NP-hard for $k \geq 2$.

(ii) Give a polynomial time k-factor approximation algorithm.

(iii) Give a linear time k-factor approximation algorithm for the special case that $w(t) = 1$ for $t \in U$.

(1+2+2 points)

Exercise 2.2. The LP relaxation of the Minimum Weight Vertex Cover Problem is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where $M \in \{0,1\}^{n \times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.
Show that the above LP relaxation of the Minimum Weight Vertex Cover Problem always has a half-integral optimum solution.

(4 points)

Exercise 2.3. Consider the following local search algorithm for the unweighted version ($w \equiv 1$) Max Cut problem:
Start with an arbitrary vertex set $S \subseteq V$. Iterate the following: if a single vertex can be added to S or can be removed from S such that $|\delta(S)|$ increases, do so. If no such vertex exists, terminate and return $\delta(S)$.

(a) Prove that this algorithm is a 2-approximation algorithm. In particular, show that it runs in polynomial time.

(b) Does the algorithm always find an optimum solution for planar graphs or bipartite graphs?

(3+1 points)

Exercise 2.4. Describe an algorithm which decides if an undirected graph $G = (V, E)$ is 4-colorable in time $\mathcal{O}(|E| \cdot 2^{|V|})$.

(3 points)

Deadline: Tuesday, April 19$^{\text{th}}$, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.