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Exercise Set 1

Exercise 1.1. Prove that SATISFIABILITY remains NP-complete if each clause contains at most three literals and each variable appears at most three times, but is in P if additionally each clause contains exactly three (pairwise different) literals.

(4 points)

Exercise 1.2. Show that the following problem is NP-complete:

Instance: A directed graph G.

Question: Is there some $X \subseteq G$ such that $E(G[X]) = \emptyset$ and that for all $v \in V \setminus X$ we have $\delta^+_{G[X \cup \{v\}]}(v) \neq \emptyset$?

Hint: Use a reduction from SATISFIABILITY.

(4 points)

Definition. A decision problem is in coNP if its complement is in NP.
It is *coNP-complete* if it is in coNP and each problem in coNP polynomially reduces to it.

Exercise 1.3. Show that it is coNP-complete to decide if two given logical formulas are logically equivalent.

(4 points)

Definition. For $\tau \leq 1$, a *τ-approximation algorithm* for the maximum stable set problem is a polynomial time algorithm that computes for every undirected graph $G = (V, E)$ a stable set $S \subseteq V$ such that $|S| \geq \tau \cdot \max\{|S^*| : S^* \subseteq V \text{ is a stable set}\}$.

Exercise 1.4. Prove: If there is a $\frac{1}{2}$-approximation algorithm for the maximum stable set problem, there is also a $(1-\epsilon)$-approximation algorithm for every constant $\epsilon \in \left(0, \frac{1}{2}\right]$.

(4 points)

Deadline: Tuesday, April 12th, until 2:15 PM (before the lecture) via eCampus. LaTeX-submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). Solutions may be submitted in groups of up to 2 people.

The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/appr_ss22_ex.html`

In case of any questions feel free to contact me at puhlmann@or.uni-bonn.de.