Prof. Dr. Stephan Held
Markus Ahrens, M. Sc.

Exercise Set 10

Exercise 10.1. Let (G, H) be a pair of undirected graphs on $V(G) = V(H)$ with capacities $u : E(G) \to \mathbb{R}_+$ and demands $b : E(H) \to \mathbb{R}_+$. A *concurrent flow* of value $\alpha > 0$ is a family $(x^f)_{f\in E(H)}$ where x^f is an s-t-flow of value $\alpha \cdot b(f)$ in $(V(G), \{(v, w), (w, v) \mid \{v, w\} \in E(G)\})$ for each $f = \{t, s\} \in E(H)$, and

$$\sum_{f\in E(H)} x^f\Big((v, w)\Big) + x^f\Big((w, v)\Big) \leq u(e)$$

for all $e = \{v, w\} \in E(G)$. The Maximum Concurrent Flow Problem is to find a concurrent flow with maximum value $\alpha > 0$.

Prove that the Maximum Concurrent Flow Problem is a special case of the Min-Max Resource Sharing Problem. Specify how to implement block solvers.

(5 points)

Exercise 10.2. Show that the Vertex-Disjoint Paths Problem is NP-complete even if G is a subgraph of a track graph G_T with two routing planes. Recall that in this case G_T is a graph $G_T = (V, E)$ for some $n_x, n_y \in \mathbb{N}$ with $V = \{1, \ldots, n_x\} \times \{1, \ldots, n_y\} \times \{1, 2\}$ and $E = \{\{(x, y, z), (x', y', z')\} : |x - x'|z + |y - y'|(3 - z) + |z - z'| = 1\}$.

Hint: Consider the proof of Theorem 5.2.

(5 points)

Exercise 10.3. A posynomial function $f : \mathbb{R}^n_{>0} \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}$$

for $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function.

(b) Let f be a posynomial function with lower and upper bounds $l, u \in \mathbb{R}^n_{>0}$, $l \leq u$ on the variables. Show that each local minimum of f on the box $[l, u]$ is also a global minimum of f on $[l, u]$.

Hint: Use a logarithmic variable transformation to derive an equivalent convex problem.

(2 + 3 points)

Exercise 10.4. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ $(1 \le i \le n)$ depicted in Figure 10.1. Assume that the delay

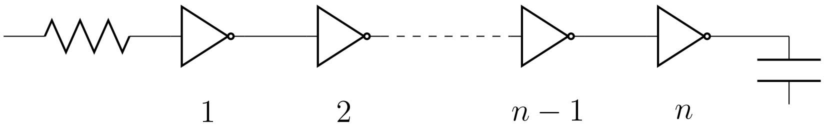

Figure 10.1: Chain of inverters.

θ_i through inverter i is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i} \qquad \text{for } 1 \le i < n-1$$

where $x = (x_1, \ldots, x_n)$, $\alpha \ge 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size x_i of the i-th inverter in a solution x of the total delay minimization problem for fixed x_1, x_n:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \le i \le n-1 \right\}.$$

(5 points)

Deadline: July 2nd, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.