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Exercise Set 9

Exercise 9.1. Consider the following algorithm to compute a rectilinear Steiner tree T for a set P of points in the plane $\mathbb{R}^2$.

Choose $p \in P$ arbitrarily;
$T := (\{p\}, \emptyset), S := P \setminus \{p\}$
while $S \neq \emptyset$ **do**
  Choose $s \in S$ with minimum $dist(s, T)$
  if $E(T) = \emptyset$ **then**
    $T := (\{p, s\}, \{\{s, p\}\})$
  else
    Let $\{u, w\} \in E(T)$ be an edge which minimizes $dist(s, SP(u, w))$
    $v := \arg\min\{dist(s, v) \mid v \in SP(u, w)\}$
    $T := (V(T) \cup \{v\} \cup \{s\}, E(T) \setminus \{\{u, w\}\} \cup \{\{u, v\}, \{v, w\}, \{v, s\}\})$
  end if
  $S := S \setminus \{s\}$
end while

In this notation $SP(u, w) \subset \mathbb{R}^2$ is the area covered by shortest paths between u and w, and $dist(s, T)$ is the minimum distance between s and the shortest path area $SP(u, w)$ of an edge $\{u, w\} \in E(T)$.

Show that the algorithm is a $\frac{3}{2}$-approximation algorithm for the Minimum Steiner Tree Problem.

(5 points)

Exercise 9.2. Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \dots, S_m \subseteq \mathbb{R}^2$ be rectangular, axis-parallel blockages. Let $S := \bigcup_i S_i$, $\mathring{S}$ denote the interior of S, and let $0 < L \in \mathbb{R}$ be a constant.

A rectilinear Steiner tree Y for T is *reach-aware* if every connected component of $E(Y) \cap \mathring{S}$ has length at most L.

(a) We define the *Hanan grid induced by* $(T, S_1, \dots, S_m)$ as the usual Hanan grid for $T \cup \{l_i, u_i \mid 1 \leq i \leq m\}$ where l_i (resp. u_i) is the lower left (resp. upper right) corner of S_i.

Prove or disprove: If there is a reach-aware Steiner tree there is always a shortest reach-aware Steiner tree for T that is a subgraph of the Hanan grid induced by $(T, S_1, \dots, S_m)$.

(b) Prove that it is $\mathcal{NP}$-hard to compute a reach-aware Steiner tree for T that has at most twice the length of an optimum solution, even with the assumption that all terminals $t \in T$ have distance at most L to unblocked area.

Hint: If there are no terminals on blockages this is not $\mathcal{NP}$-hard.

(5 points)

Exercise 9.3. Formulate the Simple Global Routing Problem as an integer linear program with a polynomial number of variables and constraints.

(5 points)

Exercise 9.4. Consider the Escape Routing Problem: We are given a complete 2-dimensional grid graph $G = (V, E)$ (i.e. $V = \{0, \dots, k-1\} \times \{0, \dots, k-1\}$ and $E = \{\{v, w\} \mid v, w \in V, \|v - w\| = 1\}$) and a set $P = \{p_1, \dots, p_m\} \subseteq V$. The task is to compute vertex-disjoint paths $\{q_1, \dots, q_m\}$ s.t. each q_i connects p_i with a point on the border $B = \{(x, y) \in V \mid \{x, y\} \cap \{0, k-1\} \neq \emptyset\}$.

Find a polynomial-time algorithm for the Escape Routing Problem or prove that the problem is NP-hard.

(5 points)

Deadline: June 25th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.