Exercise Set 8

Exercise 8.1. Given $u, v \in \mathbb{R}^2$, let

$$\mathcal{L}(u,v) := \{x \in \mathbb{R}^2 : \max\{||x-u||_1, ||x-v||_1\} < ||v-u||_1\}.$$

Let Y be a canonical tree that is full. Prove that any pair of distinct edges $(u,v), (x,y) \in E(Y)$ satisfies $\mathcal{L}(u,v) \cap \mathcal{L}(x,y) = \emptyset$.

Remark: This criterion can be used to prune prospective full components.

(5 points)

Exercise 8.2. (a) Show that the Steiner ratio for the Euclidean plane is at least $2/\sqrt{3}$.

(b) Show that the Steiner ratio for a metric space is at most $2 - 2/n$ where n is the number of terminals.

(2 + 3 points)

Exercise 8.3. Consider the following CLUSTERED RECTILINEAR STEINER TREE PROBLEM: Given a partition $T = \dot{\bigcup}_{i=1}^{k} P_i$ of the terminals ($\emptyset \neq P_i \subseteq \mathbb{R}^2, |P_i| < \infty$), find a (rectilinear) Steiner tree Y_i for each set of terminals P_i and one rectilinear, toplevel (group) Steiner tree Y_{top} connecting the embedded trees Y_i ($i = 1, \ldots, k$). The task is to minimize the total length of all trees.

Let A be an α-approximation algorithm for the RECTILINEAR STEINER TREE PROBLEM. A feasible solution to the CLUSTERED RECTILINEAR STEINER TREE PROBLEM can be found by first selecting a connection point $q_i \in \mathbb{R}^2$ for each $i = 1, \ldots, k$ and then computing $Y_i := A(P_i \cup \{q_i\})$ and $Y_{\text{top}} := A(\{q_i : 1 \leq i \leq n\})$.

(a) Show that picking $q_i \in P_i$ arbitrarily yields a 2α approximation.

(b) Prove that choosing each q_i as the center of the bounding box of P_i implies a $\frac{7}{4}\alpha$ approximation algorithm.

(c) Show that both approximation ratios above are tight.

(2 + 4 + 2 points)

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Deadline: June 18th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.