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Exercise Set 6

Exercise 6.1. Consider quadratic netlength minimization in x-dimension based on the (quadratic) Clique netmodel i.e.

$$\textsc{CliqueSQ}(N) := \sum_{\{p,q\}\subseteq N} \frac{w(N)}{|N|-1}\Big(x(p) + x(\gamma(p)) - x(q) - x(\gamma(q))\Big)^2$$

Show that CliqueSQ can be replaced equivalently by the quadratic StarSQ netmodel

$$\textsc{StarSQ}(N) := w'(N) \cdot \min\left\{\sum_{p\in N} \left(x(p) + x(\gamma(p)) - c\right)^2 | c \in \mathbb{R}\right\}$$

for an appropriate weight function w'.

(4 points)

Exercise 6.2. Prove that unless P = NP, there is no polynomial time n^α approximation algorithm for the Quadratic Assignment Problem for any $\alpha < 1$ even if $w \equiv 1$, $c \equiv 0$, $d : U \times U \to \{0, 1\}$ is metric and G is a tree.

Hint: Transformation of 4-Partition, where G is chosen as a collection of stars (one for each item) whose centers are connected to (an additional) common root vertex. U can be chosen as $|U| = |V(G)|$.

(6 points)

Exercise 6.3. Consider the *spreading LP* for $d = 2$:

$$\begin{array}{lrr}
\min & \displaystyle\sum_{e\in E(G)} w(e)\, l(e) & \\
\text{s.t.} & \displaystyle\sum_{y\in X} l(\{x,y\}) \geq \frac{1}{4}\left(|X|-1\right)^{3/2} & x \in X \subseteq V(G) \\
& l(\{x,y\}) + l(\{y,z\}) \geq l(\{x,z\}) & x,y,z \in V(G) \\
& l(\{x,y\}) \geq 1 & x,y \in V(G),\ x \neq y \\
& l(\{x,x\}) = 0 & x \in V(G)
\end{array}$$

Show that the optimum of the spreading LP is a lower bound for the cost of any 2-dimensional arrangement.

(5 points)

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Exercise 6.4. The Minimum Cut Linear Arrangement Problem is defined as follows: Given a hypergraph $G = (V, E)$ where $E \subseteq \mathcal{P}(V)$, find a bijective mapping $f : V \to \{1, ..., |V|\}$ that minimizes

$$\max_{i \in \{1,...,|V|-1\}} \left| \left\{ e \in E : \exists v, w \in e \text{ s.t. } f(v) \leq i < f(w) \right\} \right|$$

Show that this problem can be solved in $O(nm2^n)$ where $n := |V|, m := |E|$.

(5 points)

Deadline: June 4^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.