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Exercise Set 4

Exercise 4.1. Consider the following wirelength model for a $(\mathcal{C}, P, \gamma, \mathcal{N})$. For a net $N \in \mathcal{N}$,

$$\begin{aligned}\text{SmoothBB}(N) \quad := \quad & \ln\left(\sum_{p\in N} \exp\left(x(\gamma(p)) + x(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-x(\gamma(p)) - x(p)\right)\right) \\ & + \ln\left(\sum_{p\in N} \exp\left(y(\gamma(p)) + y(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-y(\gamma(p)) - y(p)\right)\right).\end{aligned}$$

Prove:

(a) $\text{BB}(N) \leq \text{SmoothBB}(N) \leq \text{BB}(N) + 4 \ln |N|$.

(b) $\text{SmoothBB}(N)$ is a convex function in $(x_C)_{C\in\mathcal{C}}$.

(Hint: it is worth simplifying the notation before proving the core mathematical property.)

(2+3 points)

Exercise 4.2. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \dots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \dots, k\}$ s.t.

$$\sum_{e=\{v,w\}\in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that f is not required to be injective.

Prove that this problem can be solved optimally by solving $k - 1$ minimum weight s-t-cut problems in digraphs with $\mathcal{O}(|V|)$ vertices and $\mathcal{O}(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s, t\} \cup C$ and

$$\begin{aligned}E_j := & \Big\{(s, v) : \exists\, w \in V \setminus C, f(w) \leq j, \{v, w\} \in E\Big\} \cup \\ & \Big\{(v, w) : v, w \in C, \{v, w\} \in E\Big\} \cup \\ & \Big\{(v, t) : \exists\, w \in V \setminus C, f(w) > j, \{v, w\} \in E\Big\}\end{aligned}$$

(5 points)

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Exercise 4.3. Consider the Standard Placement Problem on instances without blockages, where $h(C) \equiv 1 \equiv w(C)$ (unit size for $C \in \mathcal{C}$) as well as $w(N) \equiv 1$ (unit net weights for $N \in \mathcal{N}$).

Prove or disprove that this problem is NP-hard.

(5 points)

Exercise 4.4. Consider an instance of the Multisection Problem with k regions and a feasible fractional assignment. Prove that there is an integral partition which violates capacity constraints by at most

$$\frac{k-1}{k} \max \{\text{size}(C) : C \in \mathcal{C}\} .$$

(5 points)

Deadline: May 21$^{\text{th}}$, 12:15. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.