Prof. Dr. Stephan Held
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Exercise Set 3

Exercise 3.1. For a Boolean circuit C with inputs $1, \dots, n$ and arrival times $t_i \in \mathbb{N}$ $(i = 1, \dots, n)$, its delay is defined as its depth after prepending a path with t_i circuits to input i $(i = 1, \dots, n)$.

(a) Show that for n inputs with arrival times $t_i \in \mathbb{N}$ $(i = 1, \dots, n)$ there are n-ary AND, OR or XOR circuits over B_2 with delay $d \in \mathbb{N}$ if and only if

$$\sum_{i=1}^{n} 2^{t_i - d} \leq 1.$$

(b) Provide an algorithm that finds such a circuit in $\mathcal{O}(n \log n)$ time.

(3 + 2 points)

Exercise 3.2. Let $m \in \mathbb{N}$. Show that a circuit C for $f_{0,m}$ over the basis $\{\wedge, \vee\}$ with depth $D(C) \leq \log_2 m + \log_2 \log_2 m + \mathcal{O}(1)$ and size $S(C) \in \mathcal{O}(m \log m)$ can be computed in time $\mathcal{O}(m^3)$.

(6 points)

Exercise 3.3. Let T be a finite, nonempty subset of $\mathbb{R}^2$. Show that Clique can be computed in $O(|T| \log |T|)$ time where

$$\text{Clique}(T) := \frac{1}{|T| - 1} \sum_{\{(x,y),(x',y')\} \subseteq T} \left(|x - x'| + |y - y'| \right).$$

(4 points)

Exercise 3.4. Let N be a finite set of pins, and let S_p be a set of axis-parallel rectangles for each $p \in N$. We want to compute the *bounding box netlength* of N, i.e. an axis-parallel rectangle R with minimum perimeter s.t. for every $p \in N$ there is an $S \in S_p$ with $R \cap S \neq \emptyset$.

Show how to compute such a rectangle in $O(n^3)$ time where $n := \sum_{p \in N} |S_p|$.

(5 points)

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Deadline: May 14th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.