# Exercise Set 2

**Exercise 2.1.** A Boolean function $f \in B_n$ *depends essentially* on all its variables if for every $1 \leq i \leq n$ the subfunctions $f|_{x_i=0}$ and $f|_{x_i=1}$ are different.

Let $f \in B_n$ be a function that essentially depends on all its variables. Show:

(a) $S_{B_2}(f) \geq n-1$,

(b) $D_{B_2}(f) \geq \lceil \log_2 n \rceil$.

(5 points)

**Exercise 2.2.** Define a class of functions $(f_n)_{n \in \mathbb{N}}$ such that $f_n \in B_n$ and their SOP representations have size $\Omega(2^n)$.

(5 points)

**Exercise 2.3.** Let $f \in B_n$ be a Boolean function given as an oracle (i.e. for each $x \in \{0,1\}^n$ the value $f(x)$ can be computed in $\mathcal{O}(1)$ time). Show that the set $PI(f)$ of all prime implicants can be computed in $\mathcal{O}(n^2 3^n)$ time.

(5 points)

**Exercise 2.4.** Consider the following recursively defined family of Boolean functions $f_{2n,m} \in B_{2n+m+1}$ $(n, m \in \mathbb{N})$:

$$\begin{aligned}
f_{0,0}(x_0) &= x_0, \\
f_{0,m}(x_0, \dots, x_m) &= x_0 \wedge f^*_{0,m-1}(x_1, \dots, x_m) && (m \geq 1), \\
f_{2n,m}(x_0, \dots, x_{2n+m}) &= x_0 \wedge f_{2n-2,m}(x_2, \dots, x_{2n+m}) && (m \geq 0, n \geq 1),
\end{aligned}$$

where $f^*(x_1, \dots, x_n) := \neg f(\bar{x}_1, \dots, \bar{x}_n) \in B_n$ is the dual function of a function $f \in B_n$. Prove the following split equation

$$\begin{aligned}
f_{2n,2k+m+1}(x_0, x_1, \dots, x_{2n+2k+m+1}) &= f_{2n,2k}(x_0, \dots, x_{2n+2k}) \\
&\quad \wedge f^*_{2k,m}(x_{2n+1}, x_{2n+2}, \dots, x_{2n+2k+m+1}).
\end{aligned}$$

(5 points)

**Deadline:** May 7th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/chipss20_ex.html`

In case of any questions feel free to contact me at ahrens@dm.uni-bonn.de.