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Exercise Set 9

Exercise 9.1. The METRIC BIPARTITE TRAVELING SALESMAN PROBLEM is the problem of finding a Hamiltonian cycle of minimum cost in a bipartite graph H with nonnegative cost function d satisfying

$$d(a,b) + d(b,a') + d(a',b') \geq d(a,b') \text{ for } \{a,b\}, \{a',b\}, \{a,b'\}, \{a',b'\} \in E(G).$$

Prove that for any k, if there is a k-factor approximation algorithm for the METRIC BIPARTITE TRAVELING SALESMAN PROBLEM, there is also a k-factor approximation algorithm for the METRIC TRAVELING SALESMAN PROBLEM.

(4 points)

Exercise 9.2. Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$-approximation algorithm for the TSP in this special case.

Hint: You may use that a minimum weight 2-matching, i.e. a minimum weight subgraph of G in which every vertex has degree 2, can be computed in polynomial time.

(4 points)

Exercise 9.3. Consider the following algorithm for the METRIC TRAVELING SALESMAN PROBLEM:

Start with an arbitrary vertex $u \in V(K_n)$. Find a shortest edge $e = \{u, v\} \in E(K_n)$ connecting u to another vertex v. This yields a subtour $T = (u, v, u)$. Let $U := V(K_n) \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

(i) Find $w \in U$ with shortest distance to one of the nodes in T.

(ii) Add w to T between two neighbouring nodes $i, j \in T$ (by deleting the edge $\{i, j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $c(i, w) + c(w, j) - c(i, j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation.

(4 points)

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Exercise 9.4. Consider the following variant of the METRIC TSP: Given an instance of the METRIC TSP, we look for a Hamiltonian path of minimum weight (with arbitrary start- and endpoint). Give a $\frac{3}{2}$-approximation algorithm for this problem.

(4 points)

Deadline: Thursday, July 2nd until 14:15 via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (i.e. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions please contact us at approx-ss20@or.uni-bonn.de.