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Exercise Set 8

Exercise 8.1. Consider the restriction $\mathcal{P}$ of the unweighted VERTEX COVER PROBLEM to graphs where the maximum degree of every vertex is bounded by a constant B.

Let $\varepsilon > 0$. Show: If there exists a polynomial time approximation algorithm for the STEINER TREE PROBLEM with performance ratio $1 + \epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B + 1)\epsilon$.

(4 points)

Exercise 8.2. Show that the contraction lemma still holds when the edges added between terminals have lengths larger than 0. (We add parallel edges if there already is an edge.)

(3 points)

Exercise 8.3. Show that the "vertex version" of the contraction lemma is wrong:

Construct a complete graph with metric edge lengths and vertex sets A, B and C, such that

$$0 < \mathrm{mst}(A) - \mathrm{mst}(A \cup C) < \mathrm{mst}(A \cup B) - \mathrm{mst}(A \cup B \cup C),$$

where $\mathrm{mst}(X)$ for a vertex set X denotes the length of a minimum spanning tree in the graph induced by X.

(3 points)

Exercise 8.4. Consider an instance $G = (V, E)$ of the STEINER TREE PROBLEM with terminal set R and edge length $c : E \to \mathbb{R}_+$. Denote the full components of an optimum k-Steiner tree $\mathrm{SMT}_k(R)$ with $T_1^*, \ldots, T_l^*$.

(i) Suppose that $V \setminus R$ forms a stable set. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_l^*)) .$$

(ii) Suppose that all shortest paths between any two vertices in G have length 1 or 2. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_l^*)) .$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

(2+2+2 points)

Deadline: Thursday, June 25$^{\text{th}}$ 14:15, via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions feel free to contact us at approx-ss20@or.uni-bonn.de.