Exercise Set 7

Exercise 7.1. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(a) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P.

(b) Prove that this problem is NP-hard and give a 2-factor approximation algorithm.

(2+2 points)

Exercise 7.2. Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.

(Note: You may use that the Voronoi regions are computed with Dijkstra's algorithm.)

(4 points)

Exercise 7.3. Consider the following algorithm for the Steiner Tree Problem with 3 terminals v_1, v_2 and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

(i) $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log(|V|))$ time and works correctly.

(4 points)

Exercise 7.4. Give an $\mathcal{O}(n^3t^2)$ algorithm for the Steiner Tree Problem in planar graphs with all terminals lying on the outer face, where n is the number of vertices and t the number of terminals.

(Hint: Modify the Dreyfus-Wagner algorithm.)

(4 points)

Deadline: Thursday, June 18$^{\text{th}}$ until 14:15 via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (i.e. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions please contact us at approx-ss20@or.uni-bonn.de.

Note: On Thursday, 11.06.2020 there will be an online-meeting of all math students (Fachschaftsvollversammlung). All further information can be found at www.fsmath.uni-bonn.de