Exercise Set 4

Exercise 4.1. Consider the MULTIPROCESSOR SCHEDULING PROBLEM: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.
Give a $\frac{3}{2}$-approximation algorithm to the MULTIPROCESSOR SCHEDULING PROBLEM.

Hint: sort the jobs.

(3 points)

Exercise 4.2. Consider the following procedure for (unweighted) MINIMUM VERTEX COVER: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(4 points)

Exercise 4.3. Consider the MINIMUM WEIGHT VERTEX COVER PROBLEM, and recall its LP relaxation, i.e. $\min\{cx : M^T x \geq 1, x \geq 0\}$, where $M \in \{0,1\}^{n \times m}$ is the incidence matrix of an undirected graph $G = (V, E)$ and $c \in \mathbb{R}_+^{V(G)}$. Assume that you are given a coloring $\varphi : V \to \{1, \ldots, k\}$ of G. Derive a $(2 - 2/k)$-approximation algorithm from Exercise 2.4.

(3 points)

Exercise 4.4. Let G be a k-colorable graph with n vertices, where k is a constant. We define $x_k := n^{1-\frac{1}{k-1}}$ and for $2 \leq l < k$, $x_l := x_{l+1}^{1-\frac{1}{l-1}}$. For simplicity, we assume that n is chosen such that x_l is a natural number for $l \in \{2, \ldots, k\}$.

Prove that there exists a polynomial time algorithm that colors G with kx_k colors.

(6 points)

Deadline: Thursday, May 21st 14:15, via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (e.g. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions feel free to contact us at approx-ss20@or.uni-bonn.de.