Prof. Dr. Stefan Hougardy
Pietro Saccardi, Siad Daboul

Exercise Set 3

Exercise 3.1. Consider the following variant of the k-Center problem:

Instance: A complete graph $G = (V, E)$, metric edge weights $d : E(G) \to \mathbb{R}$, a partition $V = C \dot\cup S$, an integer $k \in \mathbb{N}$.

Output: A set $X \subseteq S$ with $|X| \leq k$ that minimizes

$$\max_{c \in C} \left\{ \min_{s \in X} \{d(c, s)\} \right\}.$$

(i) Show that this problem does not admit a $(3-\varepsilon)$-approximation for any $\varepsilon > 0$ unless P=NP.

(ii) Give a 3-approximation algorithm.

(4+4 points)

Exercise 3.2. Show that, unless P=NP, for any α polynomially computable in the input size, there is no α-approximation algorithm for the k-Center problem if we do not require the distance function to satisfy the triangle inequality.

(2 points)

Exercise 3.3. Consider the Directed Steiner Tree Problem: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at r that contains every vertex in T.

Show that a k-approximation algorithm for the Directed Steiner Tree Problem can be used to obtain a k-approximation algorithm for Minimum Weight Set Cover.

(2 points)

Exercise 3.4. An instance of MAX-SAT is called k-satisfiable if any k of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

Hint: Some variables occur in one-element clauses (w.l.o.g. all one-element clauses are positive), set them *true* with probability a (for some constant $a \in [0, 1]$), and set the other variables *true* with probability $\frac{1}{2}$. Choose a appropriately and derandomize this algorithm.

(4 points)

Deadline: Thursday, May 14th until 14:15 via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (i.e. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions please contact us at approx-ss20@or.uni-bonn.de.