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Exercise Set 1

Exercise 1.1. Prove that SATISFIABILITY remains NP-complete if each clause contains at most three literals and each variable appears at most three times, but is in P if additionally each clause contains exactly three literals.

(4 points)

Exercise 1.2. Show that the following problem is NP-complete:

Instance: A directed Graph G.

Question: Is there some $X \subseteq G$ such that $E(G[X]) = \emptyset$ and that for all $v \in V \setminus X$ we have $\delta^+_{G[X \cup \{v\}]}(v) \neq \emptyset$?

Hint: Use a reduction from SATISFIABILITY.

(4 points)

Exercise 1.3. Show that the following problem is NP-complete:

Instance: An undirected graph G. Vertex pairs $(s_1, t_1), \dots, (s_k, t_k)$.

Question: Are there vertex disjoint paths $P_1, \dots, P_k$, where P_i is a $s_i - v_i$ path for $i = 1, \dots, k$?

(4 points)

Definition. For $\tau \leq 1$, a *τ-approximation algorithm* for the maximum stable set problem is a polynomial time algorithm that computes for every undirected graph $G = (V, E)$ a stable set $S \subseteq V$ such that $|S| \geq \tau \cdot \max\{|S^*| | S^* \subseteq V \text{ is a stable set}\}$.

Exercise 1.4. Prove: If there is a $\frac{1}{2}$-approximation algorithm for the maximum stable set problem, there is also a $(1 - \epsilon)$-approximation algorithm for every $\frac{1}{2} \geq \epsilon > 0$.

(4 points)

Deadline: Friday, May 1st until 3:00 AM via eCampus. LaTeX submissions are highly encouraged, however, you can also submit a scan (i.e. obtained with a mobile phone). The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss20/appr_ss20_ex.html`

In case of any questions please contact us at approx-ss20@or.uni-bonn.de.