Exercise Set 11

Exercise 11.1. Let $G = (V, E)$ be an undirected graph with non-negative weights $w : V \to \mathbb{R}$, a set of sinks $T \subset V$, and a root vertex $r \in V \setminus T$. Additionaly, we are given required arrival times $rat : T \to \mathbb{R}$. The goal of the DELAY BOUNDED STEINER TREE PROBLEM is to compute a steiner tree S of $\{r\} \cup T$ in G with minimum weight, such that for each $t \in T$ the length of the unique r-t path in S is at most $rat(t)$. Assuming P $\neq$ NP, show that there is no $\log(|T|)$ approximation algorithm for this problem.

Hint: You may use that it is NP hard to find a $\log(n)$ approximation for SET COVER.

(5 points)

Exercise 11.2. Let $t_1, ..., t_n \in \mathbb{R}^2_{>0}$, $r := (0, 0) \in \mathbb{R}^2$, $d(x, y) := ||x - y||_1$.

(a) Show that there exists a perfect matching on $t_1, ..., t_n$ with length at most that of a steiner arborescence on $t_1, ..., t_n$ rooted in r.

(b) Describe a polynomial time algorithm that computes a $\mathcal{O}(log(n))$ approximation for a minimum length Steiner arborescence on $t_1, ..., t_n$ rooted in r, such that the length of each r-t_i path is $||r - t_i||_1$ ($i = 1, ..., n$).

(2+3 points)

Exercise 11.3. Let $\alpha > 1$ and $1 \leq \beta < 1+2/(\alpha-1)$. Construct a connected, planar graph G with $w : E(G) \to \mathbb{R}_+$ and $r \in V(G)$ that contains no spanning tree T with the following properties:

(a) For each $v \in V(G)$: $\text{dist}_{w,T}(r, v) \leq \alpha \cdot \text{dist}_{w,G}(r, v)$.

(b) For a minimum-spanning tree M: $\sum_{e\in E(T)} w(e) \leq \beta \cdot \sum_{e\in E(M)} w(e)$.

(5 points)

Exercise 11.4. Given a power consumption $P_l > 0$ for each buffer $l \in L$, extend the algorithm by van Ginneken from the lecture to obtain a PTAS for the problem of finding an assignment of buffers such that all required arrival times are met and power is minimized.

(a) First assume that for all $l \in L$, both P_l and $\frac{1}{P_l}$ are polynomially bounded in the input size.

(b) Then solve the general case using binary search.

(3+2 points)

Deadline: Tuesday, July 2$^{\text{nd}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.