Prof. Dr. Stephan Held
Benjamin Klotz, M. Sc.

Exercise Set 10

Exercise 10.1. Given a directed acyclic graph G (i.e. G might contain undirected cycles) and nonnegative edge weights, show how to compute a maximum weighted set $C \subset E(G)$ such that there is no directed path in G that contains two edges from C, using a maximum flow algorithm. Such a set C is also called an antichain.

(Given a feasible solution for an instance of the Discrete Time-Cost Tradeoff problem, it possibly can be made cheaper along antichains).

(5 points)

Exercise 10.2. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ $(1 \leq i \leq n)$ depicted in Figure 10.1. Assume that the delay

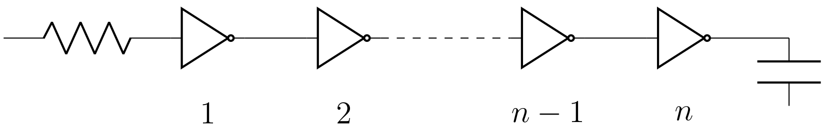

Figure 10.1: Chain of inverters.

θ_i through inverter i is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i} \qquad \text{for } 1 \leq i < n-1$$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size x_i of the i-th inverter in a solution x of the total delay minimization problem for fixed x_1, x_n:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \leq i \leq n-1 \right\}.$$

(5 points)

Exercise 10.3. A posynomial function $f : \mathbb{R}^n_{>0} \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}$$

for $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function.

(b) Let f be a posynomial function with lower and upper bounds $l, u \in \mathbb{R}^n_{>0}$, $l \leq u$ on the variables. Show that each local minimum of f on the box $[l, u]$ is also a global minimum of f on $[l, u]$.

Hint: Use a logarithmic variable transformation to derive an equivalent convex problem.

(2 + 3 points)

Exercise 10.4. The Minimum Cut Linear Arrangement Problem is defined as follows: Given a hypergraph $G = (V, E)$ where $E \subseteq \mathcal{P}(V)$, find a bijective mapping $f : V \to \{1, ..., |V|\}$ that minimizes

$$\max_{i \in \{1,...,|V|-1\}} \left| \left\{ e \in E : \exists v, w \in e \text{ s.t. } f(v) \leq i < f(w) \right\} \right|$$

Show that this problem can be solved in $O(nm2^n)$ where $n := |V|, m := |E|$.
(5 points)

Deadline: Tuesday, June 25th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.