Exercise Set 6

Exercise 6.1. Consider the *spreading LP* for $d = 2$:

$$\begin{aligned}
\min \quad & \sum_{e \in E(G)} w(e)\, l(e) & \\
\text{s.t.} \quad & \sum_{y \in X} l(\{x,y\}) \geq \frac{1}{4}\,(|X|-1)^{3/2} & x \in X \subseteq V(G)\\
& l(\{x,y\}) + l(\{y,z\}) \geq l(\{x,z\}) & x,y,z \in V(G)\\
& l(\{x,y\}) \geq 1 & x,y \in V(G),\ x \neq y\\
& l(\{x,x\}) = 0 & x \in V(G)
\end{aligned}$$

Show that the optimum of the spreading LP is a lower bound for the cost of any 2-dimensional arrangement.

(5 points)

Exercise 6.2. Provide a polynomial time algorithm for the STANDARD PLACEMENT PROBLEM restricted to instances with only one circuit.

(5 points)

Exercise 6.3. Consider the STANDARD PLACEMENT PROBLEM on instances without blockages, where $h(C) \equiv 1 \equiv w(C)$ (unit size for $C \in \mathcal{C}$) as well as $w(N) \equiv 1$ (unit net weights for $N \in \mathcal{N}$).

Prove or disprove that this problem is NP-hard.

(5 points)

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Exercise 6.4. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \dots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \dots, k\}$ s.t.

$$\sum_{e=\{v,w\} \in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that f is not required to be injective.

Prove that this problem can be solved optimally by solving $k - 1$ minimum weight s-t-cut problems in digraphs with $\mathcal{O}(|V|)$ vertices and $\mathcal{O}(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s, t\} \cup C$ and

$$\begin{aligned} E_j := &\Big\{(s, v) : \exists\ w \in V \setminus C, f(w) \leq j, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, w) : v, w \in C, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, t) : \exists\ w \in V \setminus C, f(w) > j, \{v, w\} \in E\Big\} \end{aligned}$$

(5 points)

Deadline: May 16th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.