Exercise Set 6

Exercise 6.1. Consider the spreading LP for $d = 2$:

$$\min \sum_{e \in E(G)} w(e) l(e)$$

s.t. $\sum_{y \in X} l(\{x, y\}) \geq \frac{1}{4} (|X| - 1)^{3/2}$ \hspace{1cm} x \in X \subseteq V(G)$

$l(\{x, y\}) + l(\{y, z\}) \geq l(\{x, z\}) \hspace{1cm} x, y, z \in V(G)$

$l(\{x, y\}) \geq 1 \hspace{1cm} x, y \in V(G), \ x \neq y$

$l(\{x, x\}) = 0 \hspace{1cm} x \in V(G)$

Show that the optimum of the spreading LP is a lower bound for the cost of any 2-dimensional arrangement.

(5 points)

Exercise 6.2. Provide a polynomial time algorithm for the Standard Placement Problem restricted to instances with only one circuit.

(5 points)

Exercise 6.3. Consider the Standard Placement Problem on instances without blockages, where $h(C) \equiv 1 \equiv w(C)$ (unit size for $C \in \mathcal{C}$) as well as $w(N) \equiv 1$ (unit net weights for $N \in \mathcal{N}$).

Prove or disprove that this problem is NP-hard.

(5 points)
Exercise 6.4. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \ldots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \ldots, k\}$ s.t.

$$\sum_{e = \{v, w\} \in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that $f$ is not required to be injective.

Prove that this problem can be solved optimally by solving $k - 1$ minimum weight $s$-$t$-cut problems in digraphs with $O(|V|)$ vertices and $O(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s, t\} \cup C$ and

$$E_j := \left\{ (s, v) : \exists w \in V \setminus C, f(w) \leq j, \{v, w\} \in E \right\} \cup$$

$$\left\{ (v, w) : v, w \in C, \{v, w\} \in E \right\} \cup$$

$$\left\{ (v, t) : \exists w \in V \setminus C, f(w) > j, \{v, w\} \in E \right\}$$

(5 points)

Deadline: May 16th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss19/chipss19.html

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.