Prof. Dr. Stephan Held
Benjamin Klotz, M. Sc.

Exercise Set 5

Exercise 5.1. Consider the following CLUSTERED RECTILINEAR STEINER TREE PROBLEM: Given a partition $T = \dot{\bigcup}_{i=1}^{k} P_i$ of the terminals ($\emptyset \neq P_i \subseteq \mathbb{R}^2, |P_i| < \infty$), find a (rectilinear) Steiner tree Y_i for each set of terminals P_i and one rectilinear, toplevel (group) Steiner tree Y_{top} connecting the embedded trees Y_i ($i = 1, \ldots, k$). The task is to minimize the total length of all trees.

Let A be an α-approximation algorithm for the RECTILINEAR STEINER TREE PROBLEM. A feasible solution to the CLUSTERED RECTILINEAR STEINER TREE PROBLEM can be found by first selecting a connection point $q_i \in \mathbb{R}^2$ for each $i = 1, \ldots, k$ and then computing $Y_i := A(P_i \cup \{q_i\})$ and $Y_{\text{top}} := A(\{q_i : 1 \leq i \leq n\})$.

(a) Show that picking $q_i \in P_i$ arbitrarily yields a 2α approximation.

(b) Prove that choosing each q_i as the center of the bounding box of P_i implies a $\frac{7}{4}\alpha$ approximation algorithm.

(c) Show that both approximation ratios above are tight.

(2 + 4 + 2 points)

Exercise 5.2. Prove that the STANDARD PLACEMENT PROBLEM can be solved optimally in

$$O\Big(\big((n+s)!\big)^2\big((m+n^2+k\log k)(n+k)\log(n+k)+(sn)\big)\Big)$$

time, where $n := |\mathcal{C}|$, $k := |\mathcal{N}|$, $m := |\mathcal{P}|$ and $s := |\mathcal{S}|$.

(6 points)

Exercise 5.3. Consider quadratic netlength minimization in x-dimension based on the (quadratic) Clique netmodel i.e.

$$\textsc{CliqueSQ}(N) := \sum_{\{p,q\}\subseteq N} \frac{w(N)}{|N|-1}\Big(x(p) + x(\gamma(p)) - x(q) - x(\gamma(q))\Big)^2$$

(a) Show that CliqueSQ can be replaced equivalently by the quadratic StarSQ netmodel

$$\textsc{StarSQ}(N) := w'(N) \cdot \min\left\{\sum_{p\in N} \left(x(p) + x(\gamma(p)) - c\right)^2 |c \in \mathbb{R}\right\}$$

for an appropriate weight function w'.

(b) For a fixed placement x and a single net N let $l, r \in N$ be defined as $l := \arg\min\{x(p)+x(\gamma(p)) \mid p \in N\}$ and $r := \arg\max\{x(p)+x(\gamma(p)) \mid p \in N\}$. We further define for $p, q \in N$

$$w_{pq}^{\text{B2B}} := \begin{cases} 0 & \text{if } \{p,q\} \cap \{l,r\} = \emptyset, \\ \Big|x(q) + x(\gamma(q)) - x(p) - x(\gamma(p))\Big|^{-1} & \text{else.} \end{cases}$$

Show that the CliqueSQ netlength with weights w^{B2B} equals the (linear) bounding box netlength for placement x.

(3 + 3 points)

Deadline: May 9th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.