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Exercise Set 3

Exercise 3.1. Assume unit B_2-circuit-delay and zero wire-delay.

(a) Show that for n inputs with arrival times $t_i \in \mathbb{N}$ $(i = 1, \ldots, n)$ there are n-ary AND, OR or XOR circuits over B_2 with delay $d \in \mathbb{N}$ if and only if

$$\sum_{i=1}^{n} 2^{t_i - d} \leq 1.$$

(b) Provide an algorithm that finds such a circuit in $\mathcal{O}(n \log n)$ time.

(3 + 2 points)

Exercise 3.2. Let $n = 2^k$ for $k \in \mathbb{N}$ and a, b two n-bit numbers representing $|a|, |b| \in \mathbb{N}$. Define $f^n \in B_{2n,2n}$ as $f^n(a, b) := |a| \cdot |b|$ i.e. the product of two naturals.

(a) A *bit-shift* is a multiplication by 2^i for $i \in \mathbb{N}$. Show that $|a| \cdot |b|$ can be expressed in terms of at most 3 non-bit-shift multiplications of $\frac{n}{2}$-bit numbers, 6 additions of $2n$-bit numbers, and several bit-shifts.

(b) Show $S(n) \leq 3 \cdot S(\frac{n}{2}) + cn$ and $D(n) \leq D(\frac{n}{2}) + d \cdot \log_2 n$ for constants c and d.

(c) Let $\Omega := \{\wedge, \vee, \oplus\}$. Show $S_\Omega(f^n) = \mathcal{O}(n^{\log_2 3})$ and $D_\Omega(f^n) = \mathcal{O}(\log_2^2 n)$ for circuits with fanout 2.

(1 + 3 + 3 points)

Exercise 3.3. Let (G, c, T) be an instance of the Steiner Tree Problem, G connected, $t \in T$ a terminal and $k \in \mathbb{N}$ with $k \geq 1$.

(a) For two valid lower bounds lb_a and lb_b, define $\max(\mathrm{lb}_a, \mathrm{lb}_b)$ by

$$\max(\mathrm{lb}_a, \mathrm{lb}_b)(v, I) := \max\Big(\mathrm{lb}_a(v, I), \mathrm{lb}_b(v, I)\Big).$$

Show that $\max(\mathrm{lb}_a, \mathrm{lb}_b)$ also defines a valid lower bound.

(b) Prove that $\mathrm{lb}_{\mathrm{BB}}(v, I) := \mathrm{BB}(\{v\} \cup I)$ is a valid lower bound for instances of the Rectilinear Steiner Tree Problem.

(c) Show that $\mathrm{lb}_{\mathrm{mst}}(v, I) := \frac{\mathrm{mst}(\{v\} \cup I)}{2}$ defines a valid lower bound.

(d) Define $\mathrm{lb}_k(v, I) := \max\Big\{\mathrm{smt}(J) \;\Big|\; t \in J \subseteq I \cup \{v\}, |J| \leq k + 1\Big\}$ if $t \in I$ and $\mathrm{lb}_k(v, I) := 0$ otherwise. Show that lb_k is a valid lower bound.

(3 + 1 + 1 + 3 points)

Deadline: April 25th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/chipss19.html`

In case of any questions feel free to contact me at klotz@or.uni-bonn.de.